Technical Section: Vibration and Shock Isolation

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1.0 FUNDAMENTALS OF VIBRATION AND SHOCK

1.1 What Is Vibration?

Mechanical vibration is a form of oscillatory motion. It occurs in all forms of machinery and equipment. It is what you feel when you put your hand on the hood of a car, the engine of which is running, or on the base of an electric motor when the motor is running. Perhaps the simplest illustration of a mechanical vibration is a vertical spring loaded with weight (W), as shown in Figure 1. In this position, the deflection of the spring from its free state is just sufficient to counterbalance the weight W. This deflection is called the *STATIC DEFLECTION* of the spring. The position in which the spring is at rest is No. 1. The spring is then slowly extended to position No. 2 and released. The elastic force moves the block W upward, accelerating up to the mean position and then decelerating moving further up. The uppermost position of the weight (position No. 3) is at the same distance from position No. 1 as position No. 2, but in the opposite direction. The subsequent motion of the weight as a function of time, if there is only negligible resistance to the motion, is repetitive and wavy if plotted on a time scale as shown by line 1 in the graph. This simple model exhibits many of the basic characteristics of mechanical vibrations. The maximum displacement from the rest or mean position is called the *AMPLITUDE* of the vibration. The vibratory motion repeats itself at regular intervals (A_1, A_2, A_3). The interval of time within which the motion sequence repeats itself is called a *CYCLE* or *PERIOD*. The number of cycles executed in a unit time (for example, during one second or during one minute), is known as the *FREQUENCY*. The *UNITS OF FREQUENCY* are 1 cycle/sec or 1 Hertz (Hz) which is standard. However, "cycles per minute" (cpm) are also used, especially for isolation of objects with rotating components (rotors) which often produce one excitation cycle per revolution which can be conveniently measured in cpm. When, as in Figure 1, the spring-weight system is not driven by an outside source, the vibration is a *FREE VIBRATION* and the frequency is called the *NATURAL FREQUENCY* of the system, since it is determined only by its parameters (stiffness of the spring and weight of the block).

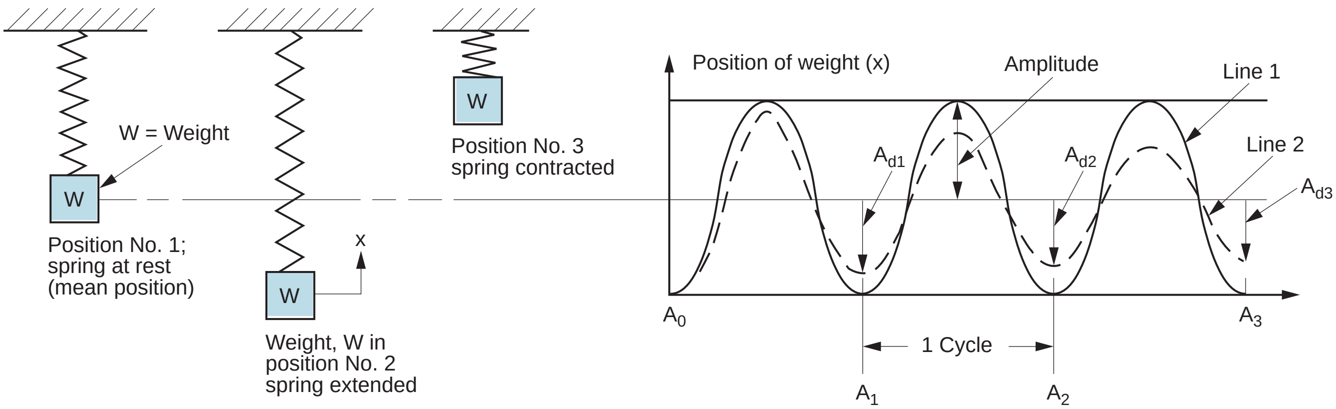


Figure 1 Free Vibrations of a Simple Vibratory System

In general, vibratory motion may or may not be repetitive and its outline as a function of time may be simple or complex. Typical vibrations, which are repetitive and continuous, are those of the base or housing of an electric motor, a household fan, a vacuum cleaner, and a sewing machine, for example. Vibrations of short duration and variable intensity are frequently initiated by a sudden impulsive (shock) load; for example, rocket upon takeoff, equipment subject to impact and drop tests, a package falling from a height, or bouncing of a freight car. In many machines, the vibration is not part of its regular or intended operation and function, but rather it cannot be avoided. Vibration isolation is one of the ways to control this unwanted vibration so that its adverse effects are kept within acceptable limits.

1.1.1 Damping

The vibratory motion as a function of time as shown in Figure 1 (line 1) does not change or fade. The elastic (potential) energy of the spring transforms into motion (kinetic) energy of the massive block and back into potential energy of the spring, and so on. In reality, there are always some losses of the energy (usually, into thermal energy) due to friction, imperfections of the spring material, etc. As a result, the total energy supporting the vibratory motion in the system is gradually decreasing (dissipated), thus diminishing the intensity (amplitude) of the spring excursions, as shown by line 2 in Figure 1 ("decaying vibration"). This phenomenon is called *DAMPING*, and energy-dissipating components are called *DAMPERS*, Figure 2. The rate of decay of amplitude in a system with damping is often characterized by *LOGARITHMIC* (or *LOG*) *DECREMENT* δ defined as

$$\delta = \log (A_n/A_{n-1}), \qquad (1)$$

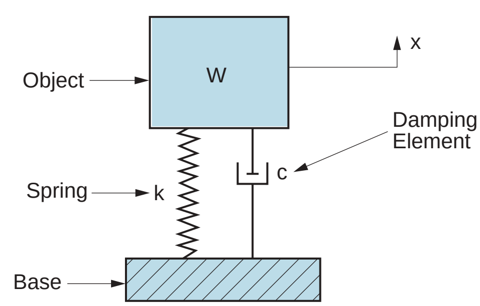


Figure 2 Simple Vibratory System with Damping

where A_n and A_{n-1} are two sequential amplitudes of the vibratory process. In many cases δ can be assumed constant during the decaying vibratory process. Although the cycles of the damped motion as shown by line 2 in Figure 1 are not fully repetitive, the number of cycles in a unit of time is still called *FREQUENCY.*

1.2 What Is Shock?

Shock is defined as a *TRANSIENT* condition whereby kinetic energy is transferred to a system in a period of time which is short, relative to the natural period of oscillation of the system. Shock usually contains a single impulse of energy of short duration and large intensity which results in a sudden change in velocity of the system undergoing shock. The principles involved in both vibration and shock isolation are similar. However, differences exist due to the steady-state nature of vibration and the transient nature of shock. Shock may occur in an infinite variety of ways and can be very complex. The simplest form is a single impulse of extremely short duration and large magnitude. Figure 3 [5] shows the most commonly employed pulse shapes used in test specifications.

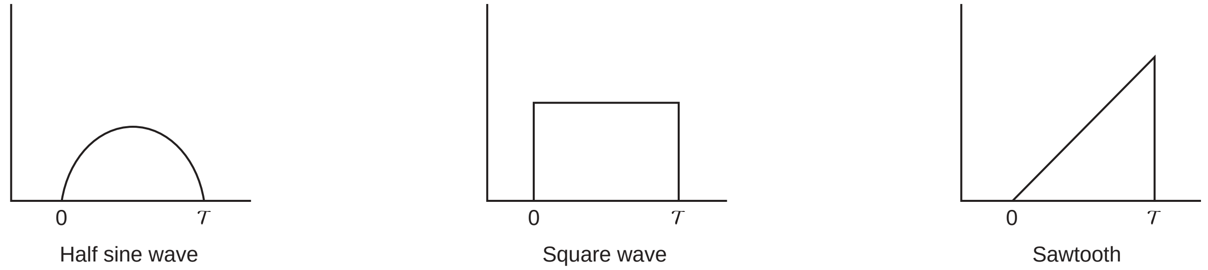


Figure 3 Basic Pulse Shapes

The reduction in shock severity, which may be obtained by the use of isolators, results from the storage of the shock energy within the isolators and its subsequent release into a "smoother" vibratory process, over a longer period of time (at the natural frequency of the spring-mass system) and/or from dissipation of the shock energy (its transformation into thermal energy). However, the energy storage can only take place by a generally large deflection of the isolator.

Inasmuch as a shock pulse may contain frequency components ranging from very low to very high, it is not possible to avoid excitation of vibratory process of the isolator/mass system with its natural frequency. On the other hand, if the duration of the shock pulse is short, the response of the system may not have serious consequences. Figure 4 [5] demonstrates the comparative response of a spring mass system to a rectangular pulse whose duration is greater than the natural period of the vibratory system (I) and to a relatively short impulsive-type shock (II).

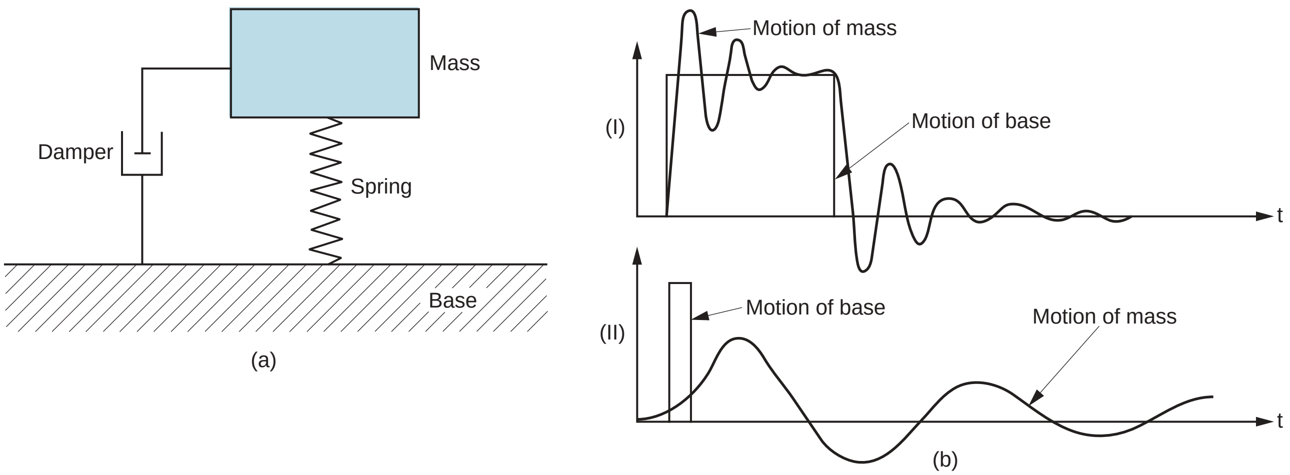


Figure 4 Response of System in Figure 2 to Rectangular Pulses of Varying Duration

1.3 What Is Noise?

Sound is a vibration of air. The air in this case is an elastic member. The vibrations of the air have a frequency and an intensity (loudness). The frequency can be expressed in cycles per second or cycles per minute. The audible frequencies range from about 20 Hz to about 18,000 Hz, although some human ears are more sensitive and may have a somewhat broader range. Some sounds are desirable and pleasant for some people, such as music. Unwanted/objectionable sounds represent *NOISE.* Intensity or loudness of noise is measured in decibels (dB). The decibel is a measure of the sound pressure in relation to a standard or reference sound (.0002 microbars, which is the threshold of hearing for sounds for many people). The sound/noise loudness in dB is equal to 20 times the common logarithm of this ratio. Typical values of sound pressure level in dB are shown in Tables 1a and 1b.

Table 1a: SOUND PRESSURE LEVELS (SPL) FROM TYPICAL NOISE SOURCES

SPL dB	Effect	Source
180	Impairs Hearing	Rocket engines
160	Impairs Hearing	Jet aircraft engines
140	Pain	Jet aircraft engine
120	Threshold of pain	Thunder, artillery
110	Deafening	Nearby riveter, elevated train
100		Boiler factory, loud street noise
90	Very Loud	Noisy factory, unmuffled truck
80		Police whistle, noisy office
70	Loud	Average street noise, average radio
60		Average factory, noisy home
50	Moderate	Average coversation, average office
40		Quiet radio, quiet home or private office
30	Faint	Average auditorium, quiet conversation
20		Rustle of leaves, whisper
10	Very Faint	Soundproof room
0		Threshold of hearing

From: *Marks' Standard Handbook for Mechanical Engineers*, Sixth Edition, McGraw Hill Book Co. Inc. New York, 1958, Section 12, p. 153; and "How to Specify Audible Noise" by E.A. Harris and W.E. Levine, *Machine Design* Nov. 9, 1961, p. 168.

Various industrial operations and related noise levels recorded at distances of from one to three feet from machine. **

Table 1b: VALUES OF SOUND AND NOISE INTENSITY

Machine	Overall Sound Pressure Level
Grinder (portable)	90-100 decibels
Drop hammer	100-105 decibels
Lathes	80-90 decibels
Punch press	95-105 decibels
Riveting guns	95-105 decibels
Sander (portable)	80-95 decibels
Screw machine	90-100 decibels
Sewing machines	90-100 decibels
Wood saw	95-100 decibels

** From: "Acoustical Enclosures Muffle Plant Noise" by S. Wasserman and A. Oppenheim, *Plant Engineering*, January 1965

1.4 Principles of Vibration Isolation

In discussing vibration isolation, it is useful to identify the three basic elements of all vibrating systems: the *object to be isolated* (equipment unit, machine, motor, instrument, etc.); the *isolation system* (resilient isolation mounts or isolators); and the *base* (floor, base plate, concrete foundation, etc). The isolators (rubber pads, springs, etc.), are interposed between the object and the base. They are usually much smaller than the object and the base.

If the object is the source of vibration, the purpose of vibration isolation is to *reduce the force* transmitted from the object to the base.

If the base is the source of vibration, the purpose of isolation is to *reduce the vibratory motion* transmitted from the base to the object, so that vibratory displacements in the work zone (between the tool and the part in a precision machine tool, the measuring stylus and the measured part in a coordinate measuring machine, the object and the lens in a microscope, etc.) do not exceed the allowable amounts. That is, probably, the most common case (protecting delicate measuring instruments and precision production equipment from floor vibrations, transportation of vibration-sensitive equipment, etc.).

In both cases, the principle of vibration isolation is the same. The isolators are resilient elements. They act as a time delay and as a source of temporary energy storage, which evens out the force or motion disturbance on one side of the vibration mounts and transmits, if properly selected, a lesser disturbance to the other end (to the base in case of force isolation, to the object in case of motion isolation).

A judicious design of the vibration isolation system insures that this effect is achieved. Conversely, a **poorly designed isolation system**, not having proper frequency characteristics, **can be worse than no isolation at all.**

In addition to its function as a time delay and source of temporary energy storage, vibration mounts can also function as energy dissipators or absorbers. This effect is usually produced by the damping characteristics of materials, viscous fluids, sliding friction, and dashpots, although in general these may or may not be part of the isolator. The damping, or energy-dissipating effect of an isolator may be negligible or substantial depending on the application. The main purpose of isolator damping is to reduce or to *attenuate* the vibrations as rapidly as possible. Damping is particularly important at certain frequencies which cause RESONANCE. This occurs when the natural frequency of the object on isolators comes close to the vibration frequency of the source. For example, if an electric motor runs at 3600 rpm, then the object-isolator natural frequency of 3600 cycles per minute (60 Hz) corresponds to the resonance condition. If a machine operates near resonance, or has to pass through a resonant speed in order to attain the operating speed, damping is important in alleviation of the vibration buildup.

In summary, a good vibration mount functions as a time delay, temporary energy absorber and to some extent as an energy dissipator, or damper. The engineering design of a vibration mount consists in identifying the characteristics of the source of the vibration, the mechanical characteristics of the equipment and the determination of the mount characteristics, in order to achieve a specified degree of vibration reduction.

1.5 Principles of Noise Reduction

A good vibration isolation system is reducing vibration transmission through structures and thus, radiation of these vibration into air, thereby reducing noise.

There are many ways to reduce noise. One of the most practical and effective may be the use of vibration mounts. As a general rule, a well-designed vibration isolator will also help reduce noise. In the case of panel flutter, for example, a well-designed vibration mount could reduce or eliminate the noise. This can be achieved by eliminating the flutter of the panel itself, or by preventing its transmission to ground, or by a combination of the two. The range of audible frequencies is so high that the natural frequencies of a vibration mount can usually be designed to be well below the noise-producing frequency.

In order to reduce noise, try to identify its sources; e.g., transformer hum, panel flutter, gear tooth engagement, rotor unbalance, etc. Next, identify the noise frequencies. Vibration isolators for these sources designed in accordance with the guidelines for vibration and shock control may then act as barriers either in not conducting the sound, or in attenuating the vibration which is the source of the noise.

2.0 BASIC DEFINITIONS AND CONCEPTS IN VIBRATION AND SHOCK ANALYSIS

2.1 Kinematic Characteristics

COORDINATE — A quantity, such as a length or an angle, which defines the position of a moving part. In Figure 1, x is a coordinate, which defines the position of the weight, W.

DISPLACEMENT — A change in position. It is a vector measured relative to a specified position, or frame of reference. The change in x (Figure 1) measured upward, say, from the bottom position, is a displacement. A displacement can be positive or negative, depending on the sign convention, translational or rotational. For example, an upward displacement may be positive, and a downward displacement negative. Similarly, a clockwise rotation may be positive and a counterclockwise rotation negative. Units: inches, feet, meters (m), millimeters (mm), or, in the case of rotations: degrees, radians, etc.

VELOCITY — The rate of change of displacement. Units: in/sec, mph., m/sec, etc. Velocity is a vector whose magnitude is the *SPEED*. Angular velocity might be measured in radians/sec or deg/sec, clockwise or counterclockwise.

ACCELERATION — The rate of change of velocity. Units: in/sec^2, m/sec^2, etc. It is a vector and has a magnitude and direction. Angular acceleration might be measured in rad/sec^2 or deg/sec^2, clockwise or counterclockwise.

VIBRATORY MOTION — An oscillating motion; such as, that of the weight W, in Figure 1.

SIMPLE VIBRATORY MOTION — A form of vibratory motion, which as a function of the time is of the form $x = a \sin \omega t$, where a and ω are constants. The maximum displacement, a, from the mean position ($x = 0$) is the *AMPLITUDE*; the *FREQUENCY* (rate at which the motion repeats itself) is $f = \omega/2\pi$ cycles/sec, where *ANGULAR FREQUENCY* ω has the dimensions of rad/sec, and frequency f has the dimensions of reciprocal time; e.g. reciprocal seconds 1/sec. Such motion is also called harmonic or sinusoidal motion.

PERIOD, CYCLE — The interval of time within which the motion repeats itself. In Figure 5, this is T seconds. The term cycle tends to refer also to the sequence of events within one period.

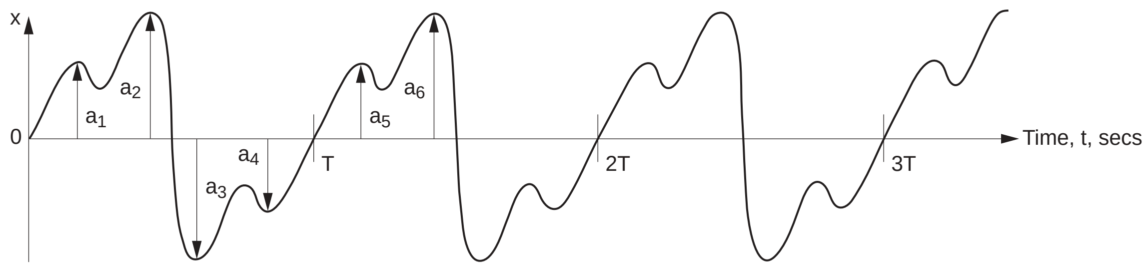


Figure 5 Periodic Motion

AMPLITUDE — Figure 5 shows time history of a vibratory motion, which repeats itself every T seconds. The maximum values of the displacement, x, from the reference position ($x = 0$) are called PEAKS. These are (a_1, a_2...). The largest of these is called the *PEAK AMPLITUDE.*

STEADY-STATE MOTION — A periodic motion of a mechanical system; e.g., a continuously swinging pendulum of constant amplitude.

STOCHASTIC or RANDOM MOTION — A motion which changes with time in a nonperiodic, possibly very complex, manner.

HARMONICS — Any motion can be considered as made up of a sum (often an infinite number) of simple harmonic motions of different frequencies and amplitudes. The lowest-frequency component is usually called the *FUNDAMENTAL FREQUENCY*; higher frequency components are called *HARMONICS*. Their frequencies are multiples of the fundamental frequency. Sometimes, components with frequencies which are fractions of the fundamental frequency (subharmonics) are significant (e.g., the "half-frequency" whirl of rotating shafts, etc.).

PULSE — Usually a displacement-time or force-time function describing a transient input into a dynamical system.

PULSE SHAPE — The shape of the time-displacement or force-displacement curve of a pulse. Typically, this might be a square wave, a rectangular pulse, or a half sine-wave pulse. In general, however, the shape can be an arbitrary function of the time.

SHOCK MOTION — A motion in which there is a sharp, nearly sudden change in velocity; e.g., a hammer blow on a nail, a package falling to the ground from a height, etc. Its mathematical idealization is that of a motion in which the velocity changes suddenly. This idealization often represents a good approximation to the real dynamic behavior of the system.

2.2 Rigid-Body Characteristics

MASS — Inertia of the body equal to its weight in lbs. or in Newtons (N) divided by the gravitational constant (g = 32.2 ft/sec^2 = 386 in/sec^2 = 9.81 m/sec^2). Unit of mass, if the weight is expressed in N, is a kilogram (kg).

CENTER OF GRAVITY (CENTER OF MASS, C.G.) — Point of support at which a body would be in balance.

MOMENT OF INERTIA — The moment of inertia of a rigid body about a given axis in the body is the sum of the products of the mass of each volume element and the square of its distance from the axis. Units are in-lb-sec^2, or in kg-m^2 for example. Moments of inertia of the standard shapes are tabulated in handbooks. If instead of mass of the element its volume is used, the result is also called a moment of inertia. Depending on the application, mass-, volume-, or area moments of inertia can be used.

PRODUCT OF INERTIA — The product of inertia of a rigid body about two intersecting, perpendicular axes in the body is the sum of the product of the mass (volumes, areas) of constituent elements and the distances of the element from the two perpendicular axes. Units are the same as for the moment of inertia. Tabulations are available in handbooks and textbooks.

PRINCIPAL AXES OF INERTIA — At any point of a rigid body, there is a set of mutually perpendicular (orthogonal) axes intersecting in the C.G. such that the products of inertia about these axes vanish. These axes are called the principal axes of inertia. In a body having axes of symmetry, the principle axes coincide with them. (An axis of symmetry is a line in the body, such that the body can be rotated a fraction of a turn about the line without changing its outline in space).

2.3 Spring and Compliance Characteristics

TENSION — When a body is stretched from its free configuration, its particles are said to be in tension (e.g., a stretched bar). The tensile force per unit area is called the *tensile stress* (Units: lbs/in^2 (psi) or Pascals, 1Pa = 1N/m^2, 1 Mega Pascal (MPa) = 10^6 N/m^2).

COMPRESSION — When a body is compressed from its free configuration (e.g., a column in axial loading), the compressive force unit per area is called the *compressive stress* (Units: lbs/in^2 or Pa).

SHEAR — When a body is subjected to equal and opposite forces, which are not collinear, the forces tend to "shear" the body; e.g., a rubber pad under parallel forces in the planes of its upper and lower faces. The shear force per unit area is called the *shear stress* (Units: lbs/in^2 or Pa). A body can be in a state of tension, compression and shear simultaneously; e.g., a beam in bending.

SPRING CONSTANT — When a helical cylindrical spring is stretched or compressed by x, the displacement x is proportional to the applied force, F (Hook's law). The proportionality constant (k) (Units: lbs/in, N/m) is called the *SPRING CONSTANT* or *STIFFNESS*, F = kx. If the spring deflects in torsion, the units of k are in-lb/rad, lb/deg, N-m/rad. Such springs are called *LINEAR SPRINGS*. More generally, the load and the displacement are not proportional (a *NONLINEAR SPRING*). In such cases stiffness is changing with the changing load and displacement, and k is the ratio of a force increment ΔF to the corresponding displacement increment Δx in the loading process. An important issue for spring materials most often used in vibration isolators, such as elastomeric (rubber) materials, wiremesh materials, etc., is influence of rate of loading on their stiffness. The stiffness constant measured at low rate of loading (frequency of load application < ~0.1 Hz) is called *STATIC STIFFNESS*, k_{st} and the stiffness constant measured at higher frequencies of load application is called *DYNAMIC STIFFNESS*, k_{dyn}. The *DYNAMIC STIFFNESS COEFFICIENT* is defined as $K_{dyn} = k_{dyn} / k_{st}$.

FORCE-DEFLECTION CHARACTERISTIC — This refers to the shape of the force-deflection curve. For the linear spring, it is a straight line through the origin of coordinates (constant k). If, for a nonlinear spring, its stiffness increases with increasing force or displacement (as in many rubber springs loaded in compression), the characteristic is called "hardening nonlinear". If it decreases with force or displacement (e.g., as in a Belleville spring), the characteristic is called "softening nonlinear".

ENERGY STORAGE — This is the area under the force-deflection curve of the spring. It represents the strain energy stored in the spring (Units: lb. in., lb. ft., N•m).

PRELOAD — A spring or other elastic element used in an isolator or in a coupling may or may not be assembled in a condition in which it has its natural, free, or unstretched length. If its assembled length is not its free length, the spring is in tension or compression even before the isolator is loaded by the object weight or the coupling is loaded by the transmitted torque. The amount of this tension or compression is called the preload. When measured in force units, it is a preload force; when measured in deflection from the free position, it is a preload deflection.

ELASTIC (YOUNG'S) MODULUS (E) AND SHEAR MODULUS (G) — These are material properties, which characterize resistance of the material to deformation in tension or in compression (E) and in shear (G). They are defined as the ratio of stress to strain, where strain is the change in length (or deformation) per unit length. E involves tensile or compressive stress/strain and G involves shear stress/strain. Units: lb/in^2, Pa. In many practical applications, especially for metals, E and G are constants within a limit of material stress known as the proportionality limit. Rubber and plastics often do not have a well-defined proportionality limit.

2.4 Damping, Friction and Energy-Dissipation Characteristics

STATIC FRICTION, SLIDING FRICTION, COULOMB FRICTION — These are all terms used for the frictional resistance for sliding of one body relative to another; e.g., a weight dragged along the floor. The frictional force is approximately proportional to the contact force between the two bodies and is opposed to the direction of relative motion. The proportionality constant f is known as the friction coefficient. If a 10 lb. weight is dragged along a horizontal floor with a friction coefficient f = 0.2, the frictional resistance is 0.2 x 10 = 2 lb. Sometimes a distinction is made between the value of the coefficient of friction when motion is just starting after a stationary condition (*STATIC FRICTION*) and its value during motion (*SLIDING* or *DYNAMIC FRICTION*). The coefficient of friction in the latter case is generally lower and changes with the motion velocity, unless it is *DRY* or *COULOMB FRICTION*, wherein the sliding friction coefficient does not depend on velocity. The motion (kinetic) energy is decreasing due to energy dissipation during a sliding process with friction. Thus, frictional connections can be used as dampers.

VISCOUS DAMPING — If, in a damper, the body moves relative to a second body, *VISCOUS DAMPING* refers to a resisting (friction) force which is proportional and opposite to the relative velocity between the two bodies. The proportionality constant is the coefficient of viscous damping, c. Units: force per unit velocity; i.e., lb/(in/sec) or N/(m/sec). Viscous damping is encountered, for example, in hydraulic dashpots and devices which squeeze a liquid through an orifice. The more viscous the fluid, the greater the damping. If c = 0.5 lb/(in/sec) and the body moves at 10 in/sec, the viscous damping force is 0.5 x 10 = 5 lb. Typical example: hydraulic door closers.

MATERIAL or HYSTERETIC DAMPING — such as damping in rubber isolators, wire mesh isolators, etc., depends on vibration amplitudes rather than on vibratory velocity. While both viscous and hysteretic damping reduce resonance amplitudes, the viscous damping spoils vibration isolation efficiency at high frequencies (when vibration amplitudes are decreasing) while the intensity of hysteretic damping automatically decreases with the decreasing amplitudes and it results in a better isolation efficiency.

CRITICAL DAMPING c_{cr} — Value of damping constant in mass-spring-damping system just sufficiently high so as to prevent vibration.

DAMPING RATIO c/c_{cr} — The ratio of the damping constant to the critical damping constant for that system. The damping ratio is related to log decrement δ as

$$\delta = 2\pi\ (c/c_{cr}). \quad (2)$$

2.5 Vibration Characteristics of Mechanical Systems

MATHEMATICAL MODEL — An idealized representation of the real mechanical system, simplified so that it can be analyzed. The representation often consists of rigid masses, springs and dampers (dashpots). The model should be sufficiently realistic so that results of the analysis of the model correspond reasonably closely to the behavior of the physical system from which it was derived.

LUMPED- AND DISTRIBUTED-PARAMETER SYSTEMS — In a lumped-parameter system, the mass, elastic spring and damping properties are separated or lumped into distinct components, each having only mass, only elasticity or only damping, but not more than one of these properties per component. In a distributed-parameter system, a component may possess combined mass, elasticity and damping, distributed continuously through the component. The latter systems represent more realistic models, but are more difficult to analyze.

DEGREES OF FREEDOM — This is the number of independent quantities (dimensions or coordinates), which must be known in order to be able to draw the mechanical system in any one position, if the fixed dimensions of the system are known. The simple mass-spring system of Figure 1 has one degree of freedom; a mechanical differential, for example, has two degrees of freedom; a rigid body moving freely in space has six degrees of freedom (three translational and three angular coordinates should be known in order to fully describe the position of the body in space).

FORCE AND MOTION EXCITATION — If a force varying in time is applied to a dynamical system, it usually is a source of vibration (e.g., centrifugal force due to an unbalanced rotor). The vibrations are then said to be due to force excitation. If, on the other hand, the foundation (or other part) of a machine is subject to a forced motion (vibration or shock), the resulting machine vibration is said to be due to motion excitation; e.g., an earthquake actuating a seismograph.

FREE VIBRATION — If the massive block in Figure 1 is moved out of its equilibrium position, and released, the system will vibrate without the action of any external forces. Such an oscillation is called a free vibration.

FORCED VIBRATION — If an external force is applied to the weight in Figure 1, which causes it to vibrate (e.g., a force varying harmonically with time), the resulting motion of the spring-mass system is called a forced vibration. If the base which supports the spring, undergoes a forced motion which in turn causes the weight to vibrate, the vibration is also forced.

RANDOM VIBRATION — Equipment may be caused to vibrate by applied forces or motions in which frequencies and amplitudes of harmonics vary in a random manner with time (e.g., wind gusts on a missile). The resulting vibration is called random.

NATURAL FREQUENCY — Whether the system is without damping or with damping, the frequency of free vibration is called the free-undamped natural frequency or the free-damped natural frequency. The natural frequency is a function of the mass and stiffness distribution in the system. For a simple-mass spring system, which is a reasonable approximation to many real mechanical systems, the natural frequency, f_n, is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} = \frac{1}{2\pi}\sqrt{\frac{g}{x_{st}}} \text{ Hz.} \qquad (3)$$

Here, k is spring constant (dynamic stiffness constant k_{dyn} should be used, see Section 2.3); W is the weight; g is the gravitational constant, 386 in/sec^2 or 9.8 m/sec^2; and x_{st} is the static deflection of the spring. The reciprocal to the natural frequency is the *NATURAL PERIOD* $T = 1/f_n$, sec. If x_{st} is expressed in cm (1 cm = 0.01 m), then the natural frequency can be conveniently found as

$$f_n \approx \frac{5}{\sqrt{x_{st}}} \text{ Hz.} \qquad (4)$$

The angular natural frequency ω_n in radians per second is

$$\omega_n = \sqrt{\frac{kg}{W}} \qquad (5)$$

Thus, flexible systems tend to have low natural frequencies and rigid systems tend to have high natural frequencies. At the same time, the natural frequency can be changed by altering the stiffness and mass distribution of the system. A system may have more than one natural frequency, in which case the lowest of these is often the most significant one. The number of natural frequencies is equal to the number of degrees of freedom of the system. Presense of damping is slightly reducing the natural frequency; The *DAMPED NATURAL FREQUENCY* is

$$f_{dn} = f_n\sqrt{1 - \left(\frac{c}{c_{cr}}\right)^2} = f_n\sqrt{\frac{1 - \delta^2}{4\pi^2}} \qquad (3a)$$

where $\delta = 2\pi\,(c/c_{cr})$
c = damping constant
c_{cr} = critical damping constant

FORCING FREQUENCY — The frequency of an external force or motion excitation applied to a vibrating system.

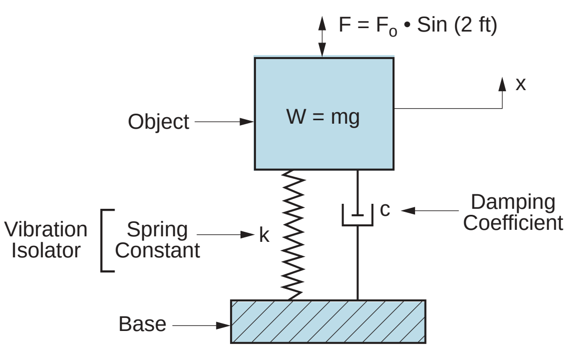


Figure 6 Simple Vibratory System Under Forced Excitation

2.5.1 Amplitude-Frequency Characteristics of Forced Vibrations

If a sinusoidal force $F(t) = F_o \sin 2\pi ft$ is acting on massive block W connected with the base by spring having stiffness k and viscous damper with resistance coefficient c, Figure 6, then sinusoidal vibration of block W is excited. If frequency f is changing but amplitude F_o is constant in a broad frequency range, then amplitude of the vibratory displacement of block W changes with frequency along an *AMPLITUDE-FREQUENCY CHARACTERISTIC*, Figure 7. Figure 7 shows plots of the displacement amplitudes vs. *FREQUENCY RATIO* f/f_n for various degrees of damping (*LOG DECREMENT* δ) in the vibratory system. The plots in Figure 7 are described by the following expression for the response amplitude A of the massive block W to the force excitation:

$$A = \frac{F_o}{k\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(2\frac{c}{c_{cr}}\frac{f}{f_n}\right)^2}} = \frac{F_o / k}{\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}} \qquad (6)$$

RESONANCE — It is seen in Figure 7 that displacement and stress levels tend to build up greatly when the forcing frequency coincides with the natural frequency, the build-up being restrained only by damping. This condition is known as *RESONANCE.*

In many cases, the forced vibration is caused by an unbalanced rotating mass, such as the rotor of an electrical motor. The degree of unbalance can be expressed as distance e between the C.G. of the rotor and its axis of rotation. The vertical component of the centrifugal force generated by the unbalanced rotor (mass M) is

$$F_{c.f.} = M\omega^2 e \sin \omega t = 4\pi^2 M f^2 e \sin 2\pi t, \quad (7)$$

where ω is angular speed of rotation in rad/sec and f is the number of revolutions per second. In case of vibration excitation by the unbalanced rotor, combining of (6) and (7) results in

$$A = \frac{4\pi^2 M f^2 e}{k\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(2\frac{c}{c_{cr}}\frac{f}{f_n}\right)^2}}$$

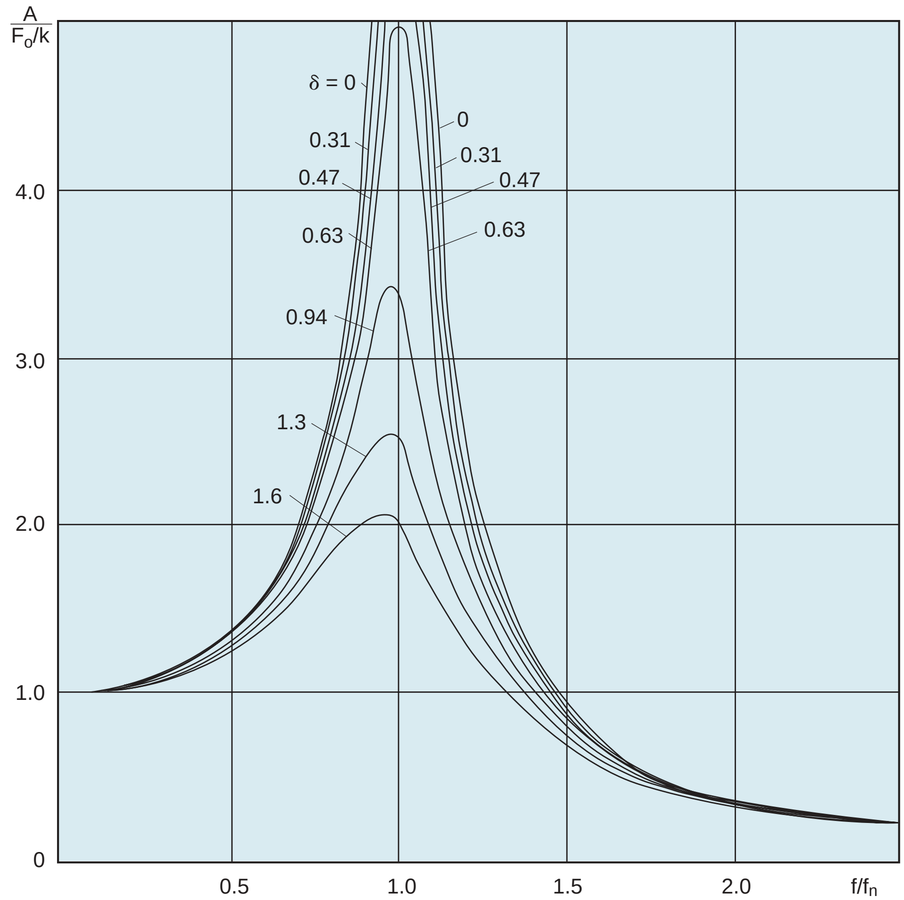


Figure 7 Amplitude-Frequency Characteristics of Massive Block Motion in Figure 6

$$= \frac{4\pi^2 M f^2 e}{k\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}} = \frac{Me}{m}\,\frac{f^2 / f_n^2}{\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}}, \qquad (6a)$$

where m is the total mass of the object. Expression (6a) is plotted in Figure 8 for several values of damping (δ).

3.0 Vibration Isolation

Although *VIBRATION ISOLATION* is a very large area of vibration control, there are two most widely used techniques of vibration isolation:

– Reduction of transmission of vibratory or shock forces from the object, in which these forces are generated, to the base; and

– Reduction of transmission of vibratory motions of the base to the work area of vibration-sensitive objects.

These techniques are similar, but also quite different. They both deal with *TRANSMISSIBILITY* or *TRANSMISSION RATIO*. There are several transmission ratios. Usually these refer to the ratios of the maximum values of the transmitted force or displacement to the maximum values of the applied force or the forced motion. The important direction of transmission is from the object to the base for the force isolation, or from the base to the object for the motion isolation.

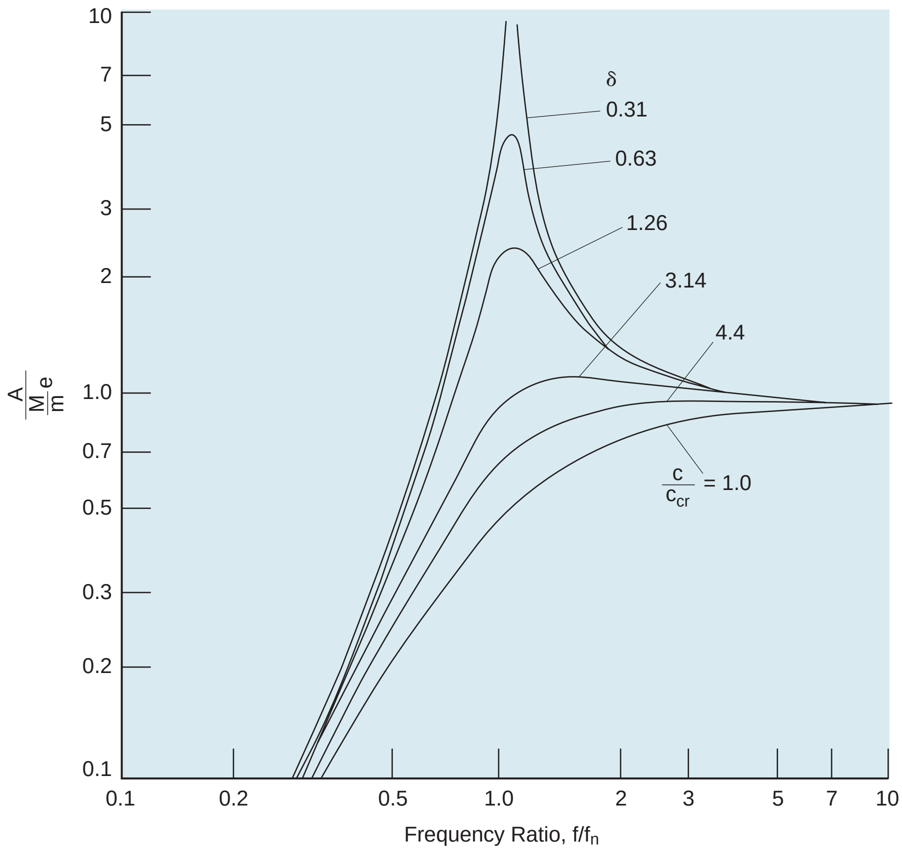


Figure 8 Amplitude-Frequency Characteristics of Massive Block Motion in Figure 6 Excited by an Unbalanced Rotor

3.1 Vibration Isolation of Vibration-Producing Products

Figure 9 shows a simplified single-degree-of-freedom model of a vibration isolation system. While in models in Figure 1 and Figure 2, the base (foundation) is shown as having infinite mass, in Figure 9 model the foundation has a finite mass m_f. If the force $F(t) = F_o \sin 2\pi ft$ is generated in the object (mass m), the force transmissibility μ_F from the object to the foundation is equal to the motion transmissibility μ_x from the foundation to the object and is expressed as

$$\mu_F = \mu_x = \frac{|F_f|}{|F_o|} = \frac{|x_1|}{|x_2|} = \frac{m_f}{m + m_f}\sqrt{\frac{1 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}} \ . \tag{8}$$

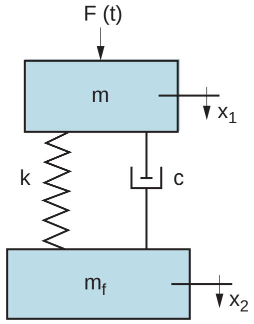


Figure 9 Dynamic Model of a Basic Vibration Isolation System

This expression (for $m_f = \infty$) is plotted in Figure 10 which shows that "isolation" of the force source or the condition of $\mu_F < 1$ develops at frequencies greater than $f = 1.41f_n$ and fast improving with further increasing of the frequency ratio f/f_n. The maximum transmissibility occurs at the resonance when the frequency ratio $f/f_n = 1$. At resonance ($f = f_n$), the transmissibility at not very high damping is expressed as

$$(\mu_F)_{max} = (\mu_x)_{max} \approx \frac{m_f}{m + m_f}\frac{\pi}{\delta} \ . \tag{9}$$

While increasing of damping is beneficial at and around the resonance, the isolation at high frequencies deteriorates with increasing damping δ. This effect must be considered in designing the isolation system for a given application. Still, a reasonable increase of damping is important since it makes the system more robust if subjected to inevitable spurious excitations. Also, the higher damping improves behavior of the system if the object generates forces in a broad frequency range; e.g., as unbalanced motor(s) generating continuously changing excitation frequency during its acceleration phase. It should be considered that the transmissibility curves in Figure 10 are plotted for viscous damping in the isolators. Damping in elastomeric and wire-mesh (or cable) elements is different, so-called hysteretic damping. This latter type of damping does not affect the preresonance and the resonance behavior of the system, but demonstrate only a minimum deterioration of the isolation at high frequencies even for highly-damped isolators (more in [1]).

As mentioned before, the goal of vibration isolation of vibration-sensitive objects from the base vibration is to reduce relative vibratory displacements in the work zone. Transmissibility of the base motion into the relative vibrations $\theta = x_1 - x_2$ is (for any value of m_f):

$$\mu_{rel} = \frac{|\theta|}{|x_2|} = \frac{\dfrac{f^2}{f_n^2}}{\sqrt{\left(1 - \dfrac{f^2}{f_n^2}\right)^2 + \left(\dfrac{\delta}{\pi}\dfrac{f}{f_n}\right)^2}}. \qquad (10)$$

Expression (10) is plotted in Figure 11. It is clear that transmissibility of low frequency (as compared with the natural frequency) foundation vibrations into the relative vibrations is very small (since at low frequencies the motions are very slow and the object is moving following the vibrating foundation).

ISOLATION EFFICIENCY — Isolation is the percent of vibration force that is not transmitted through the vibration mounts and which improves with increasing frequency ratio. Isolation efficiency of 81.1% corresponding to a frequency ratio of 2.5, is generally adequate as shown in Table 2. Figure 12, the basic vibration chart, gives static deflection vs. frequency and % of vibration isolation ($1 - \mu_F$). It is useful for selection of vibration isolators/mounts and for calculations (see Section 11).

A more complete treatment of this case of vibration isolation, considering more complex and more realistic (several degrees of freedom) models is given in [1].

Table 2: VIBRATION ABSORPTION

Frequency Ratio	Vibration Absorption, Percent	Results Attained
10.0	98.9	excellent
4.0	93.3	excellent
3.0	87.5	very good
2.5	81.1	good
2.0	66.7	fair
1.5	20.0	poor
1.4	0	none
1.0	(resonance)	worse than with no mountings

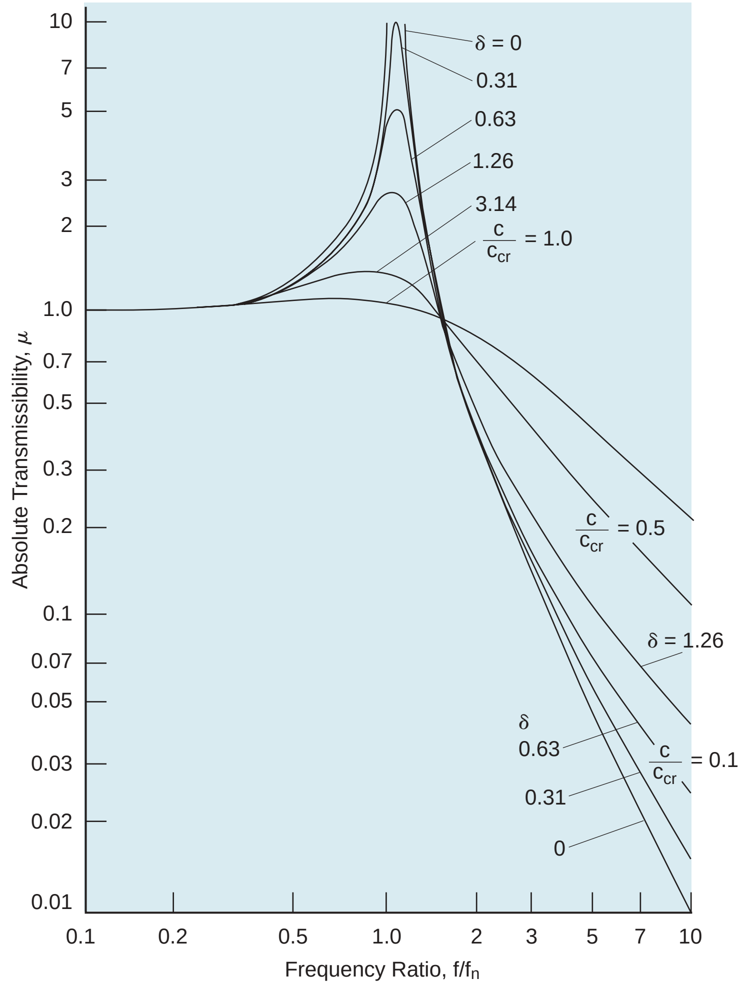


Figure 10 Force/Motion Transmissibility in Figure 9 System

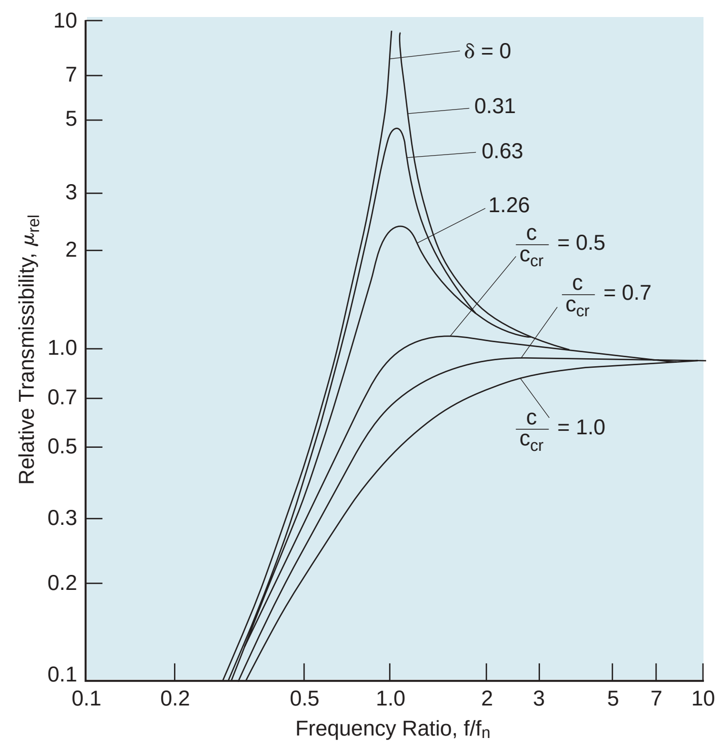


Figure 11 Transmissibility of Vibratory Base Motion to Relative Vibratory Motion in the Work Zone

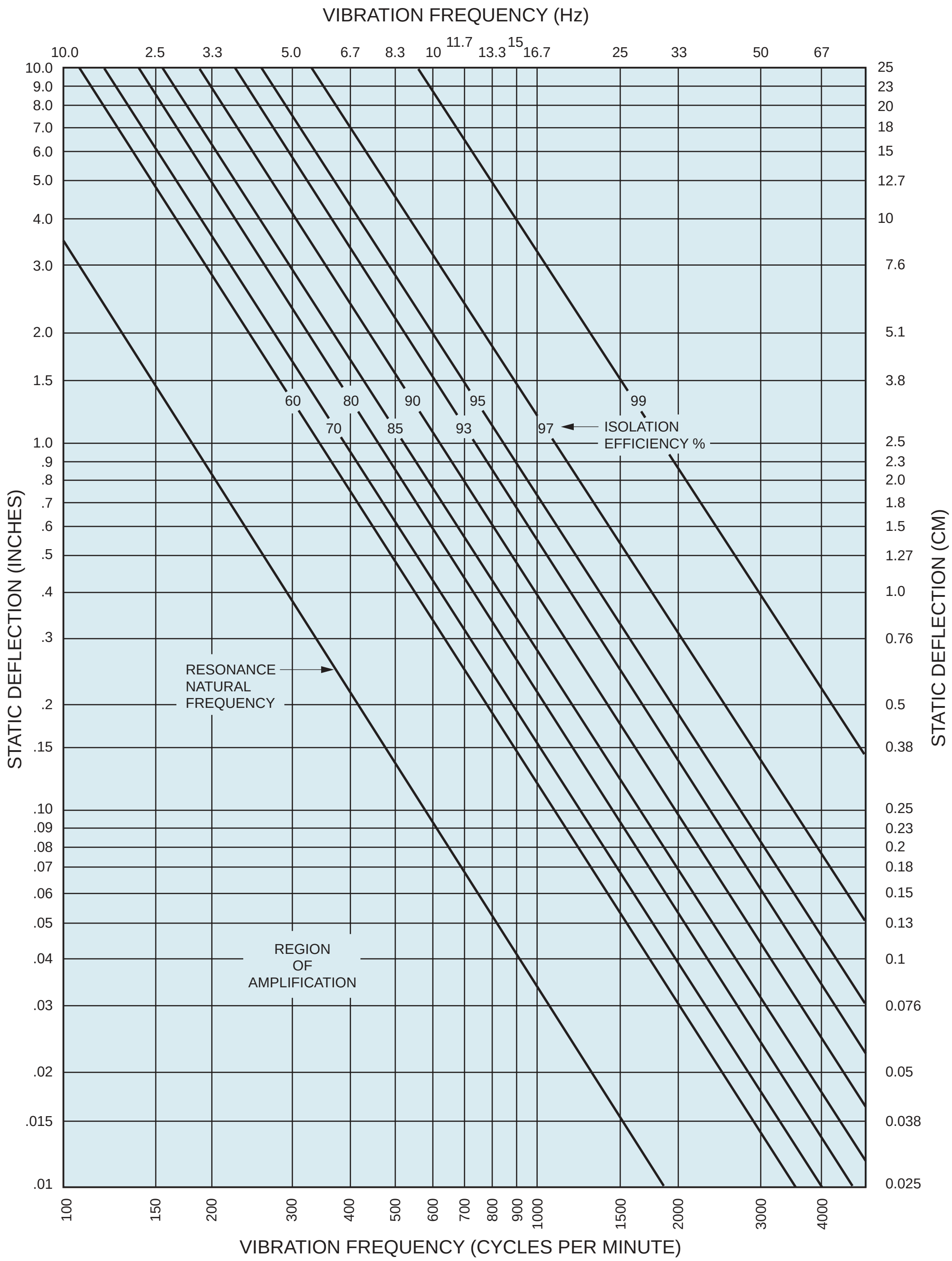


Figure 12 Vibration Frequency vs Static Deflection of Isolators vs Isolation Efficiency

3.2 Vibration Isolation of Vibration-Sensitive Objects

Since, for this group of objects, the relative vibrations in the work zone are determined by dynamic characteristics of the object itself, a model in Figure 13 should be considered. Floor (foundation) vibration $x_1 = x_{10} \sin 2\pi ft$ is transmitted through vibration isolators (stiffness k_v, damping coefficient c_v) to frame/bed of the object (mass M_B) causing its vibrations $x_B = x_{B0} \sin 2\pi ft$. The work zone of the object is between the frame/bed and its "upper unit", mass M_u (e.g., tool head of a machine tool or illumination unit of a photo-lithography tool). Stiffness k_m and damping coefficient c_m describe structural dynamic characteristics of the object, whose structural natural frequency is

$$f_m = \frac{1}{2\pi}\sqrt{\frac{k_m (M_u + M_B)}{M_u M_B}} \ . \qquad (11)$$

T1 Technical Section

Accordingly, transmissibility of the vibratory motion of the foundation into the work zone can be expressed as a product of (transmissibility μ_x of the foundation motion X_1 to the frame motion from expression (7) where $x_2 = X_1$, $x_1 = X_B$; and $m = M_B$) and (transmissibility of the frame motion X_B to the relative motion X_{rel} in the work zone μ_{rel} from expression (8) where $x_2 = X_B$, $\theta = X_{rel}$, and $f_n = f_m$ from expression (11)). This operation is illustrated in Figure 14.

In Figure 14, the plot (a) is maximum intensity a_o of floor vibration (displacements amplitudes compounded from numerous on-site measurements). It is shown in [1] that for a majority of manufacturing plants $a_o \approx 2.5\mu m$ in the 4-30 Hz range and is much smaller outside of this range for vertical floor vibrations, and $a_o \approx 2.0\ \mu m$ in the 4-20 Hz range and much smaller outside of this range for horizontal floor vibrations. For high precision facilities, the levels of allowable floor vibrations are recommended by BBN plots in Figure 15. The next plot (b) in Figure 14 illustrates transmissibility from the floor to the object frame for three cases: a - the object installed on rigid mounts (e.g., jack mounts or rigid isolator mounts); b - the object installed on softer, isolating mounts (lower f_n) with the same degree of damping (height of the resonance peak) as the mounts in a; c - the same f_n as in b, but greater damping. The third plot (c) illustrates transmissibility from the frame of the object into its work zone; f_m is the structural natural frequency of the object. The bottom plot shows the product of the previous three plots. An installation is considered successful if the vibration amplitude in the work zone does not exceed the allowable amplitude Δ_o.

It can be seen that a rigid installation results in two peaks of the relative vibration amplitude, which often exceed the tolerance. Both peaks are reduced by using soft isolator mounts: the second one due to reduced transmissibility at high frequencies per expression (6), and the first one due to lower sensitivity of the object structure to lower resonance frequency of the object on softer isolating mounts. It is clear, that increasing damping also results in reduced relative vibrations. Accordingly, the requirement for an adequate vibration isolation of a vibration-sensitive object is formulated not as a required upper limit of the natural frequency f_n, but as a required upper limit of the "Isolation Criterion" Φ,

$$\Phi = \frac{f_n}{\sqrt{\delta}}\,. \tag{12a}$$

The magnitude of this criterion can be calculated if vibration sensitivity of the object in the frequency range of interest is measured and its tolerance is assigned, see [1]. The object is properly isolated if

$$\Phi < \sqrt{\frac{\Delta_o f^2}{\pi X_f \mu_f}}\,, \tag{12b}$$

where Δ_o is the maximum tolerated vibratory displacement in the work zone of the object, X_f is the maximum amplitude of floor vibration with frequency f; μ_f is the transmissibility into the work zone at frequency f (ratio of relative vibration amplitude in the work zone to amplitude of the object frame vibration at frequency f). According to this criterion widely validated by practical applications, stiffness of isolators for a given installation can be increased (usually, a very desirable feature) if the isolators have higher damping.

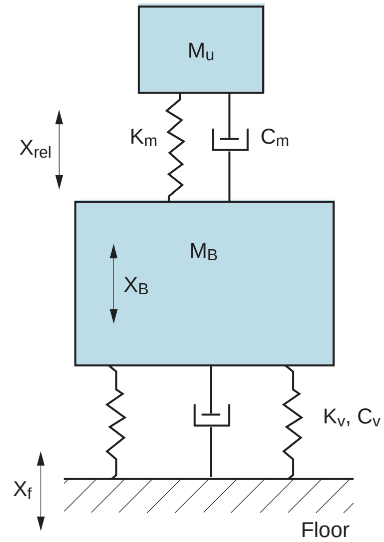
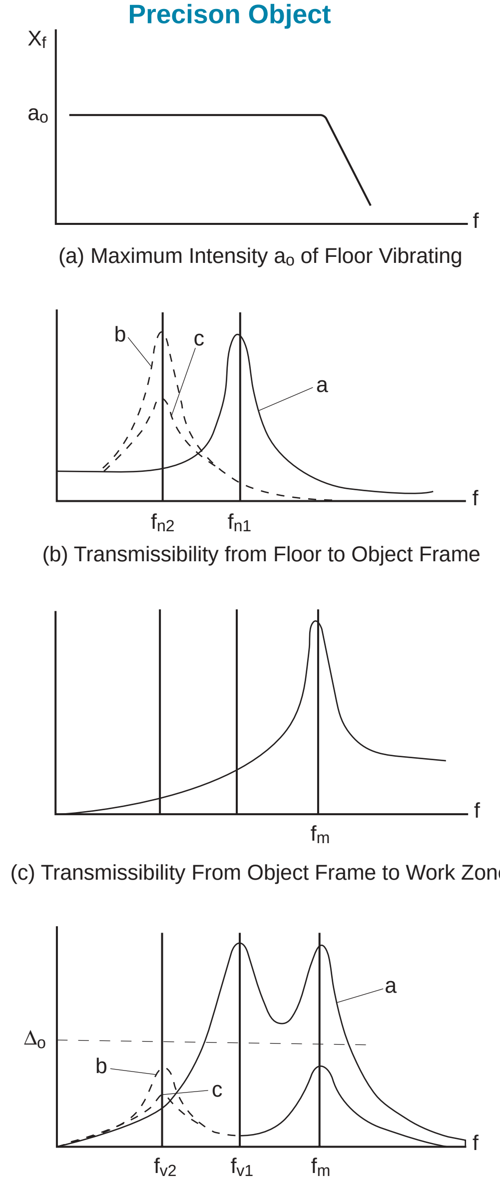


Figure 13 Two-Mass Dynamic Model for Vibration Sensitivity of Precison Object



(a) Maximum Intensity a_o of Floor Vibrating

(b) Transmissibility from Floor to Object Frame

(c) Transmissibility From Object Frame to Work Zone

(d) Resultant Transmissibility (Product of (a), (b) & (c))

Figure 14 Model of Vibration Transmission from Floor to Work Zone

Thus, while vibration isolation of the force-producing objects requires reducing natural frequency in accordance with nomogram in Figure 12, isolation of a vibration-sensitive object can be successful even when some part of the system is at resonance, provided that the natural frequency of the isolation system and its damping are properly selected. Vibration isolation in the latter case is greatly simplified if structural stiffness and structural natural frequency of the vibration-sensitive object are enhanced.

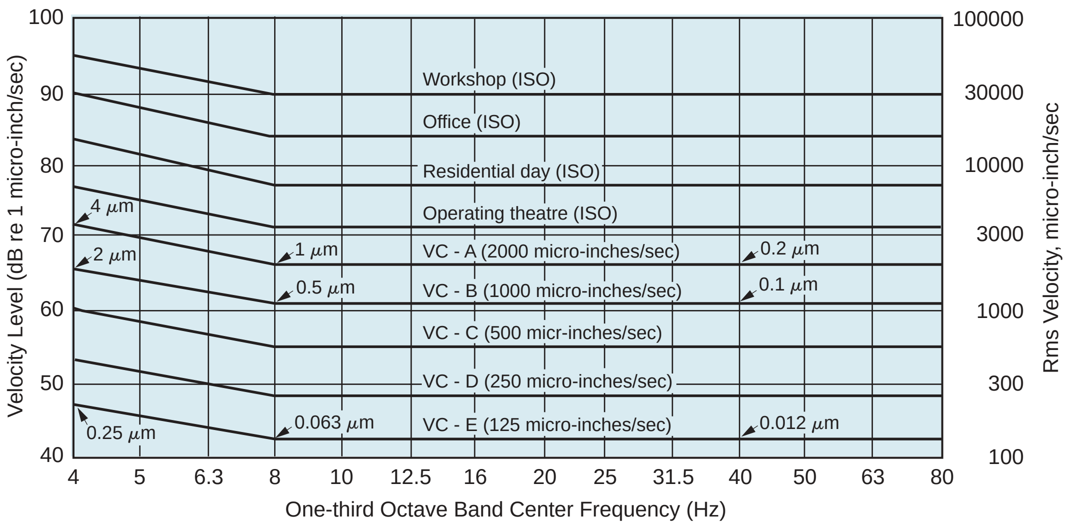


Figure 15 BBN Vibration Criteria (VC) for Installation of Precision Equipment

3.3 Shock Isolation

The information in this section has been taken from [2] with permission of the publisher.

It is often necessary to determine the effectiveness of a shock isolator as well as the magnitude of the acceleration experienced by elements of the protected equipment. Figure 16, similar to Figure 13, describes the system experiencing a velocity shock as illustrated by the displacement-time curves of Figure 17.

The displacement of equipment (y) supported by isolators and subjected to a velocity shock (V) is expressed by the following equation:

$$y = V\left(1 - \frac{1}{2\pi f_y}\sin 2\pi f_y t\right) \qquad (13)$$

where $f_y = \dfrac{1}{2\pi}\sqrt{\dfrac{k_y}{m_y}}$ is the natural frequency, Hz, of the elastic system

consisting of chassis (m_y) and isolator (k_y). Double differentiation of equation (13) yields the acceleration experienced by the equipment chassis during shock. This is designated the transmitted acceleration and is expressed as:

$$\ddot{y}_0 = 2\pi f_y V \qquad (14)$$

The units of acceleration $\ddot{y}_0$, are linear distance (inches, m, etc) per second per second. This equation can be expressed another way, using more convenient engineering units, as:

$$\text{Transmitted Shock} = \frac{\ddot{y}_0}{g} = \frac{2\pi f_y V}{386} = \frac{f_y V}{61.4}\,, \qquad (15)$$

where: V = shock velocity change, in/sec.
f_y = natural frequency of isolator, Hz.
$\ddot{y}_0/g$ = maximum acceleration experienced by chassis, expressed as a dimensionless multiple of the acceleration due to gravity.

Thus, the maximum acceleration of the chassis during shock, is directly proportional to the magnitude of the velocity change and to the natural frequency of the isolator. Figure 18 is a graphic representation of the maximum transmitted acceleration computed from Equation (15).

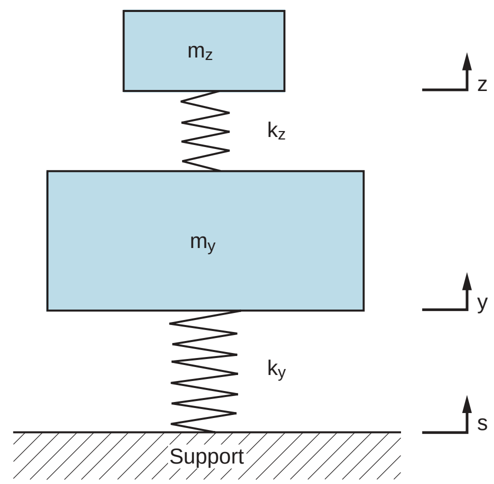


Figure 16 Schematic Representation of Equipment, Comprised of Chassis my and Element $m_z k_z$, Mounted Upon Isolator k_y.

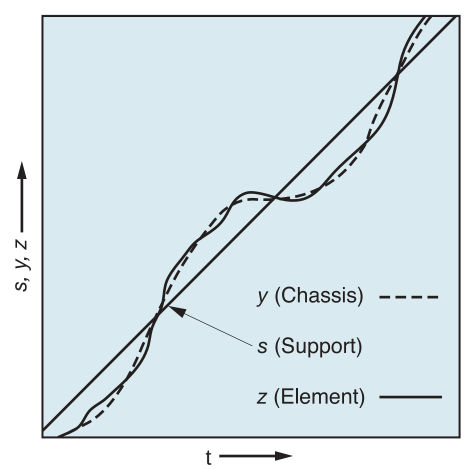


Figure 17 Displacement-Time Curves for Support, Chassis, and Element of Equipment (Inelastic Impact)

The maximum acceleration experienced by the chassis of the mounted equipment, as indicated in Figure 18, should not be confused with the maximum acceleration experienced by various elements of the equipment. The latter is equal to the product of the maximum chassis acceleration y_0 and the amplification factor A_0, which is defined as the ratio of the maximum acceleration of the element ($\ddot{z}_0$) to the maximum acceleration of the chassis ($\ddot{y}_0$) and is given by:

$$A_0 = \frac{\ddot{z}_0}{\ddot{y}_0} \qquad (16)$$

In the absence of damping, A_0 is a function only of the element's natural frequency (f_z) and the isolator's natural frequency (f_y). For an undamped system, shock transmissibility (T_s) is related to the amplification factor (A_0) as follows:

$$T_s = A_0 \left(\frac{f_y}{f_z} \right) \qquad (17)$$

where shock transmissibility (T_s) is the ratio of the maximum acceleration of the mass element, m_z, to the maximum acceleration of the same element which would occur if the isolator's spring constant, k_y, were infinitely rigid.

Using values for the amplification factor A_0 as determined in [3], and plotted for a range of values of damping ratio, shock transmissibility can be determined for a damped system as shown in Figure 19. The damping between m_z and m_y is assumed to be constant at one percent critical damping (δ = 0.063). However, wide variations in the degree of damping have little effect on the results. Figure 20 gives the amplification factor A_0 for the system shown in Figure 16 when the support experiences velocity shock as illustrated in Figure 17. The factor A_0 is the ratio of the maximum acceleration of mass m_z to the maximum acceleration of mass m_y.

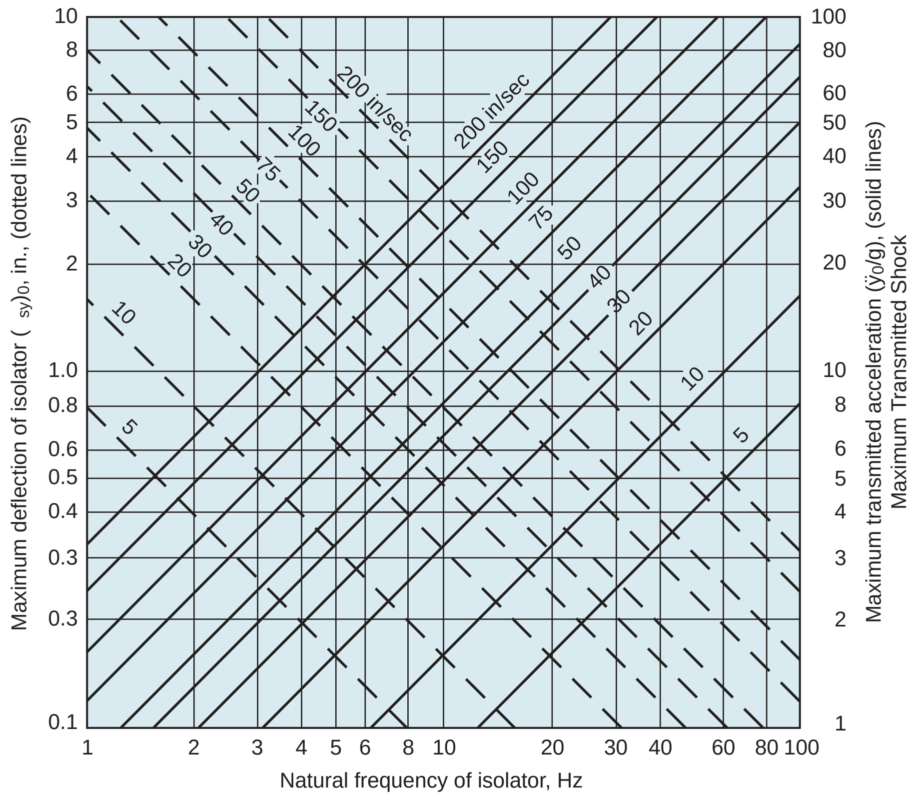


Figure 18 Maximum Acceleration of Chassis m_y and Maximum Deflection of Linear Isolator k_y Shown in Figure 16, When Support Experiences Velocity Shock as Illustrated in Figure 17.

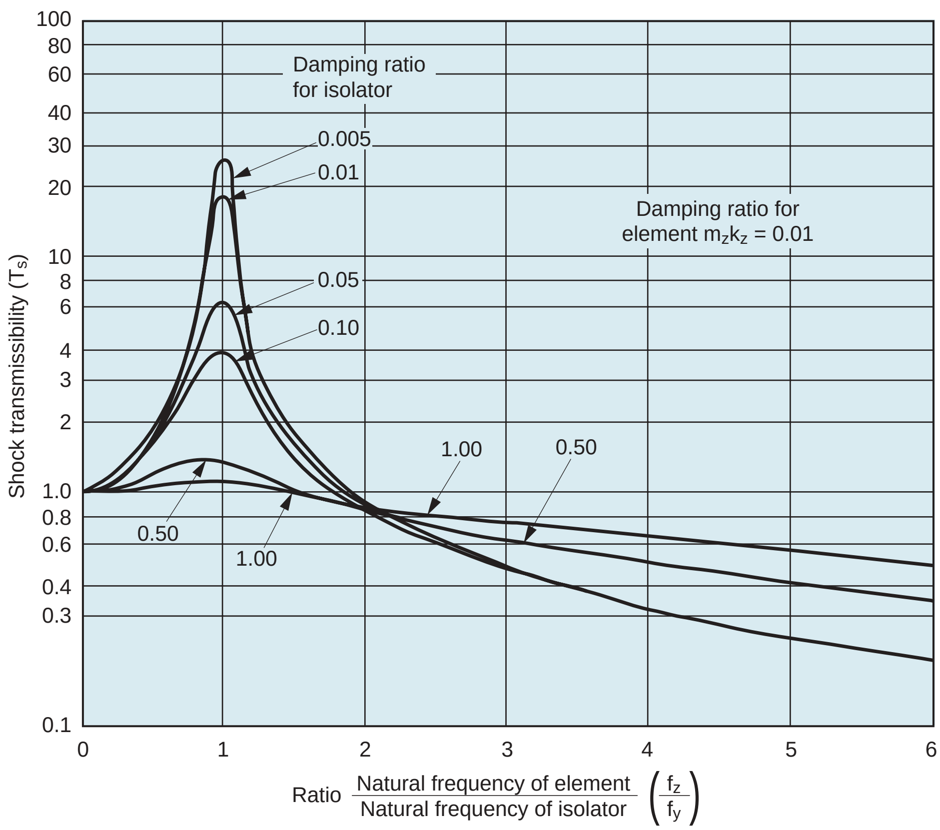


Figure 19 Shock Transmissibility for System Shown in Figure 13, When Subjected to Velocity Shock as illustrated in Figure 17 [3].

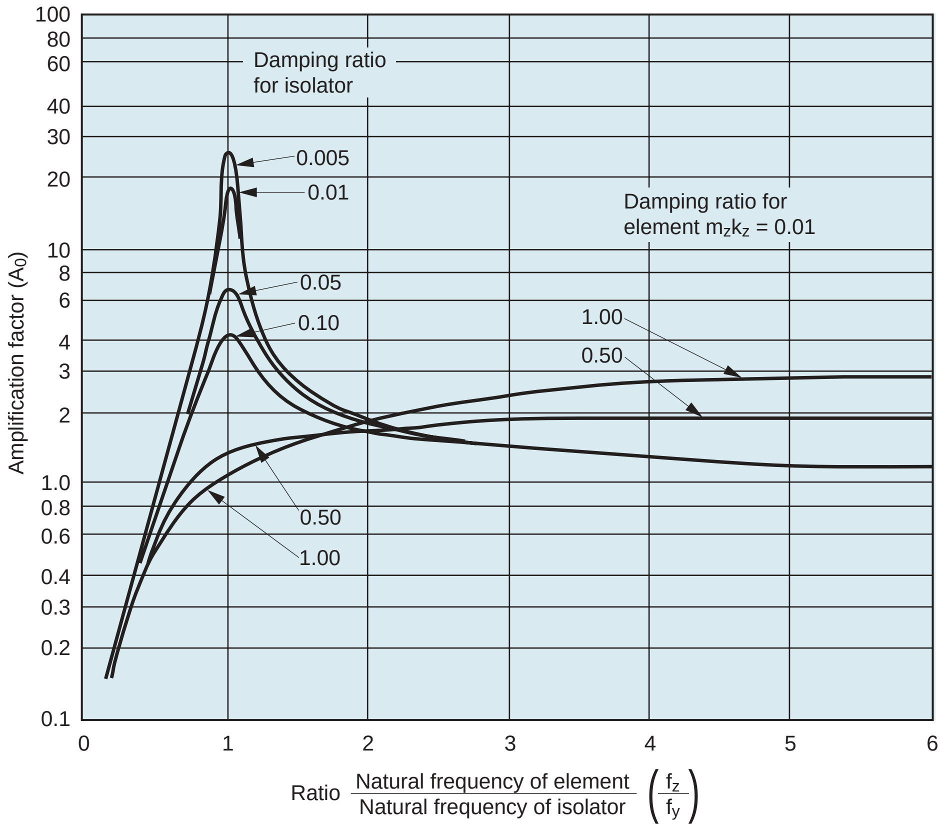
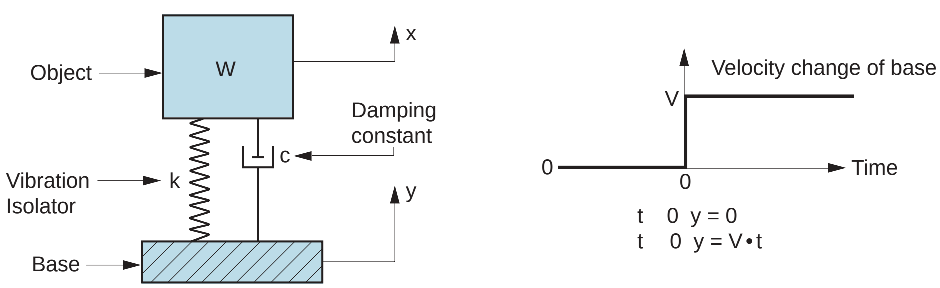


Figure 20 Amplification Factor for System Shown in Figure 16 When Subjected to Velocity Shock as Illustrated in Figure 17

3.3.1 Shock Motion of Base (Base Suddenly Stops or Accelerates)

The time history of the sudden acceleration process of the base in Figure. 21(a) is shown in Figure 21(b). The analytical results taken from [3] are also applicable to the object (equipment unit) dropping from a height onto a hard surface.



(a) System **(b) Motion of Base**

Figure 21 Vibration Isolation System for Object W (a) Subjected to Shock Motion of Base with Time History (b)

If: V = sudden velocity change of base, in/sec or m/sec

c/c_{cr} = $\delta/2\pi$ = damping ratio where δ is log decrement

f_n = undamped natural frequency of system, Hz

g = gravitational constant, 386 in/sec^2 = 9.81 m/sec^2

d_{max} = max. isolator deflection, measured from equilibrium position, in. or m

d_{st} = static isolator deflection = W/k, in. or m

a_{max} = maximum acceleration of object, in/sec^2 or m/sec^2

then, for $0 \leqslant c/c_{cr} \leqslant 0.2$ or $0 \leqslant \delta \leqslant 1.25$,

$$\frac{d_{max}}{d_{st}} = \frac{a_{max}}{g} = \frac{2\pi f_n(1 - c/c_{cr})}{g} \tag{18}$$

Figure 22 illustrates Equation (18). When the damping is small, maximum force transmitted to equipment is very nearly kd_{max}.

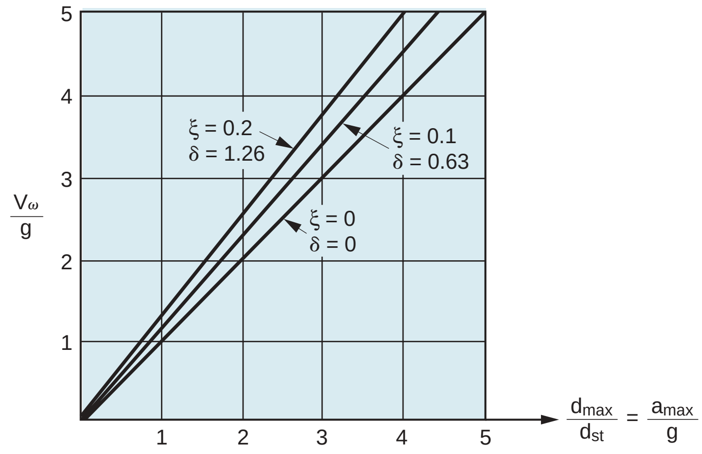
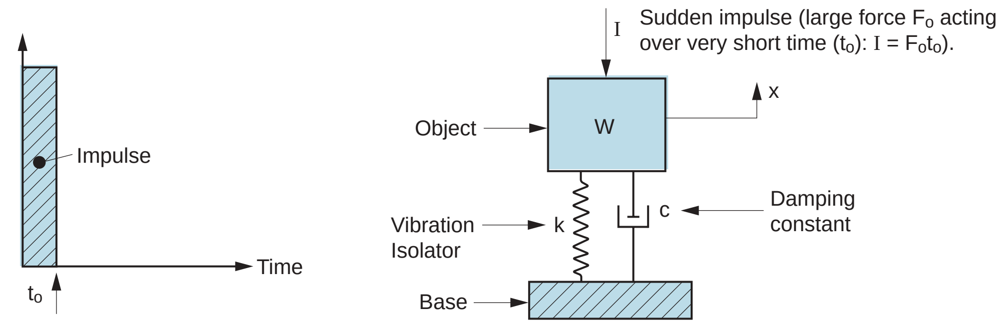


Figure 22 Shock Effect at Different Damping Values

3.3.2 Sudden Impact on Equipment [3]



(a) FORCE TIME CURVE OF AN IMPULSE

(b) SYSTEM

Figure 23 Vibration Isolation System of Object W (b) Subjected to Sudden Impact on the Object with Time History (a)

Sudden impact, or a sharp blow is characterized by a large force (F_o) acting for a short period of time (t_o) as shown in Figure 23(a). For practical purposes, suddenness is taken to mean that t_o is small in comparison with the natural period of vibration of the system in Figure 23(b). The *impulse*, I, is defined as the area under the force-time curve; i.e.,

$$I = F_o t_o \text{ lb-sec or kg m/sec} \tag{19}$$

Application of impulse I results in a sudden downward velocity V of the object,

$$V = Ig/W. \tag{20}$$

The maximum isolator deflection and the maximum acceleration of the object can be obtained by substituting V into Equation (18).

4.0 NONLINEARITIES

The equations previously given for transmissibilty (Section 3.1) make certain assumptions which may not always be valid. For example, it is assumed that the damping is viscous or *linear* (resistance to relative motion is proportional to the relative velocity). The assumption greatly simplifies the analysis. However, the damping provided by wire mesh is a combination of localized frictional losses by individual wires and hysteresis in the cushion itself. Damping in elastomeric materials has similar characteristics. In practical terms, this means that the damping is a function of displacement in addition to velocity, and the terms describing the damping in the equations of motion are *nonlinear*. At resonance, where the displacement is large, the damping is high. In the isolation band, where displacement is small, the damping is negligible. This condition gives the best of both worlds as damping is only desirable under resonance conditions. Thus, the idealized curves in Figure 10 are on the conservative side since they show deterioration of isolation in the high frequency (after resonance) range.

A second assumption is that the flexible members or mounts behave as linear springs. This again is not strictly true as many mounts behave as hardening springs to a lesser or greater extent, depending on material of their flexible elements (e.g., proportion of mesh cushion in the wire-mesh mounts) and/or on design features of rubber flexible elements. As the term suggests, the stiffness increases with load/displacement. This property has the useful effect of increasing the dynamic load-carrying capability of the mounts. Consider the Equation (3) for the natural frequency of a simple spring-mass system. As can be seen, increasing the weight load (mass) of the isolated object reduces the natural frequency. But if the stiffness is increasing as well (as in the case of a hardening spring) then the ratio k/m is less dependent on the mass of the object and the mount can be used in a wider load range. Some vibration isolators are designed with their stiffness proportional to the weight load,

$$k = AW = Amg, \tag{21}$$

where A is a proportionality constant. For such mounts the natural frequency is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} = \frac{1}{2\pi}\sqrt{\frac{AWg}{W}} = \frac{1}{2\pi}\sqrt{Ag} = \text{const.} \tag{22}$$

Accordingly, such vibration isolators are called *CONSTANT NATURAL FREQUENCY (or CNF) ISOLATORS*. This means that a mount will give the same degree of isolation for a broad load range, with the ratio of upper load limit and lower load limit up to and exceeding 20:1 [1]. An example of a CNF isolator is Ring Mount V10Z47M in this catalog.

Besides the convenience of using the same isolators for widely different objects, CNF isolators have many other advantages. The tolerance on stiffness of constant stiffness (linear) isolators with rubber flexible elements is usually about ±17%. Such wide tolerance leads to a need for greater safety factors in order to achieve the required degree of isolation, and thus to softer isolators. The soft isolators are undesirable since they may result in a shaky installation. CNF isolators, on the other hand, are very robust and variation of rubber hardness due to production tolerances do not influence the natural frequency significantly [1]. Other advantages of CNF isolators are addressed below in Section 6.0.

The other way in which a stiffening spring affects the dynamic performance of a system is to make the natural frequency "input sensitive". As the amplitude increases, so does the displacement. Consequently, that stiffness increases as well. The natural frequency (f_n) increases also. Figure 24 [5] shows a comparison between the way frequency f_n changes with amplitude for a linear spring (a) and a hardening spring (b). As can be seen, with a hardening spring, f_n increases with amplitude. Without going into the mathematical treatment, it should be appreciated that the actual responses for various inputs will be as shown in Figure 25 [5]. It can be seen that the resonant point actually changes with different inputs. A softening spring is added for comparison.

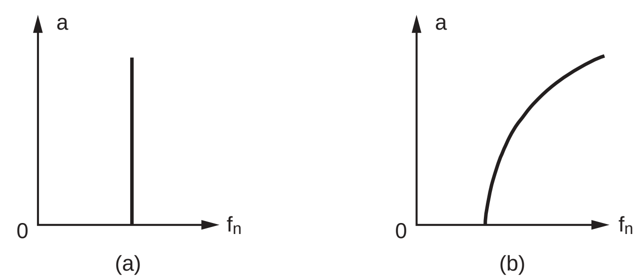


Figure 24 Amplitude of Linear (a) and Hardening (Nonlinear) (b) Springs as a Function of f_n

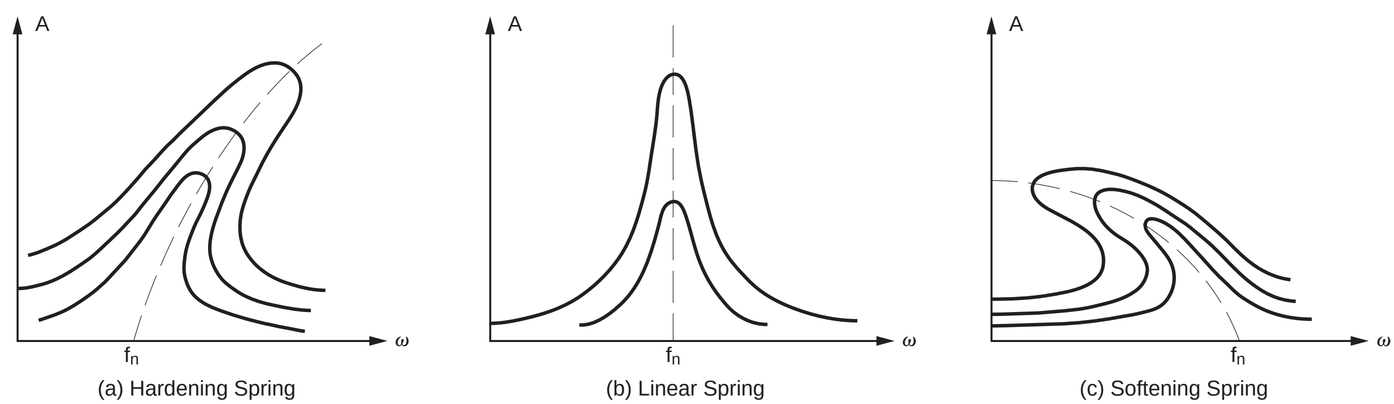


Figure 25 Typical Resonance Curves for Various Levels of Excitation

T1 Technical Section

Another property of mesh mounts is demonstrated by Figure 26 [5]. As can be seen, in practice there is a sudden sharp drop from the resonant point, ensuring that isolation is achieved almost immediately. However, it is again safer to assume that isolation does not begin until $\sqrt{2}\, f_n$ is achieved.

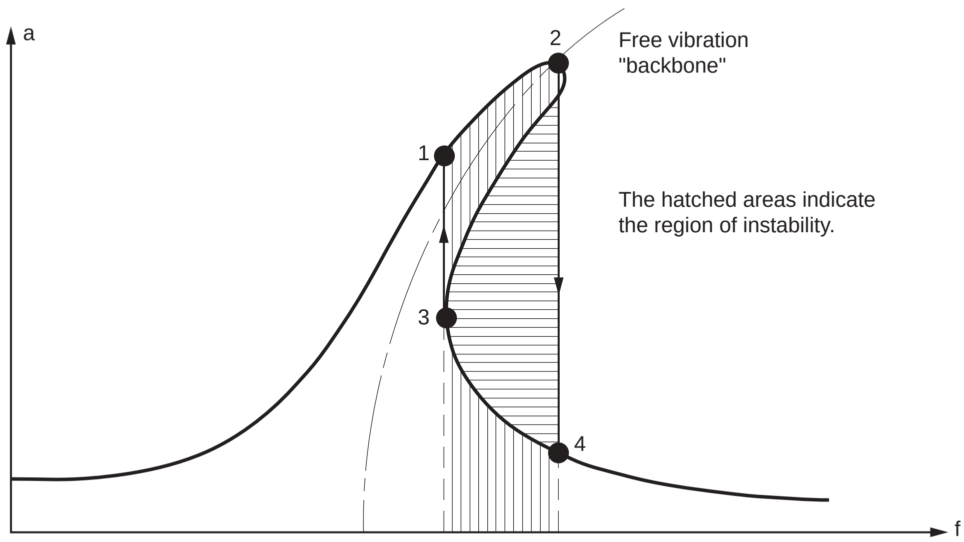


Figure 26 Theoretical Frequency Response Curve for a Hardening Spring Type Resonant System

5.0 MULTIDEGREE OF FREEDOM SYSTEMS, COUPLED MODES

Figure 27 demonstrates that there are six independent ways in which a body can move; i.e., it has SIX DEGREES OF FREEDOM. The reader must be aware from this that there is a potential of six independent natural frequencies, as well as possible coupled modes of vibration.

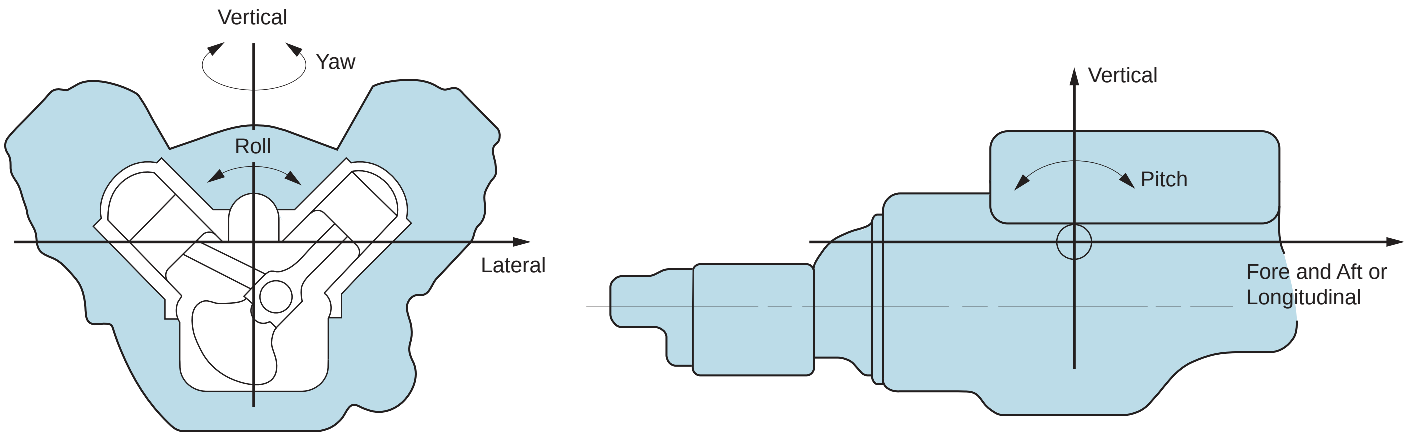


Figure 27 Degrees of Freedom of a Solid Body

The coupling concept can be illustrated on the example of a simpler "planar" system shown in Figure 28, which shows a mass supported by springs and constrained so that it can move only in the plane of the drawing [5]. Such a system has three coordinates which fully describe its configuration: translational coordinates x and y, and angular coordinate α. If the system is symmetrical about axis y, then when excited by a sinusoidal force F_y, in the vertical direction along the axis of symmetry, the object will behave as previously shown (Figure 1), namely by vibrating in the vertical (y) direction. However, if the force vector does not coincide with the axis of symmetry, then the vertical force would excite vibratory motions not only in the y-direction, but also in x and α directions. When the mass is excited by a horizontal force F_x, both horizontal (ÿ) or longitudinal mode and pitching (α) vibratory motions are excited. These modes are said to be coupled when vibrations of one mode can be stimulated by a vibratory force or displacement in another. Coupling modes are in most cases undesirable. For example, many vibration-sensitive objects have the highest vibration sensitivity in a horizontal direction, while the floor vibrations are often more intense in the vertical direction. Coupling between the vertical and horizontal directions can be avoided by using vibration isolating mounts at each mounting point whose stiffness is proportional to the weight load acting on this mount (CNF mount) [1].

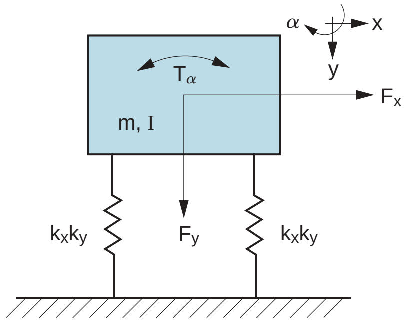


Figure 28 Planar (Three-Degrees-of-Freedom) Vibration Isolation System

6.0 STATIC LOAD DISTRIBUTION CALCULATION

In order to calculate the weight distribution between the mounting points, the position of the CENTER OF GRAVITY (C.G.) has to be determined first. It is a simple task only for an axisymmetrical object. Position of the C.G. can be obtained by computation or experiment. The computational approach is feasible in most cases to the manufacturer who has all relevant drawings containing the data on mass distribution inside the object. The experiment is suggested by the definition of the C.G. as the point of support at which the body will be in equilibrium. For example, a small object can be supported on a peg; when in equilibrium, a vertical line drawn through the peg will pass through the C.G. Unfortunately, this method is applicable only to small objects. For large objects, such as machine tools, the object is mounted, for the C.G. location purposes, onto three load cells LC1, LC2, LC3, as shown is the plane view in Figure 29. If the weight loads as sensed by these load cells are W_1, W_2, W_3, respectively, then coordinates of the C.G. are as follows:

$$\left.\begin{aligned} x_{C.G.} &= \frac{x_1W_1 + x_2W_2 + x_3W_3}{W_1 + W_2 + W_3}\ ; \\ y_{C.G.} &= \frac{y_1W_1 + y_2W_2 + y_3W_3}{W_1 + W_2 + W_3}\ . \end{aligned}\right\} \quad (23)$$

After the C.G. position is known, weight distribution between the mounting points should be calculated. Such a calculation can be rigorously performed only for the case of an object with three mounting points (a statically-determinate problem). Unfortunately, only a relatively small percentage of objects requiring vibration isolation are designed with the "three point" mounting arrangement. If the number of the mounting points is greater than three, the accuracy of weight distribution calculations is suffering, unless the mounting surface of the floor is flat and horizontal and the mounting surface of the object is also flat. The tolerance on the "flatness" requirement should be a small fraction of the projected static deformations x_{st} of the selected vibration isolators.

For example, if the vertical natural frequency of the isolated object is f_n = 20 Hz, then, from Equation (4), x_{st} = 0.0625 cm or 0.625 mm.

Similarly, for f_n = 10 Hz, x_{st} = 2.5 mm, and
for f_n = 5 Hz, x_{st} = 10 mm.

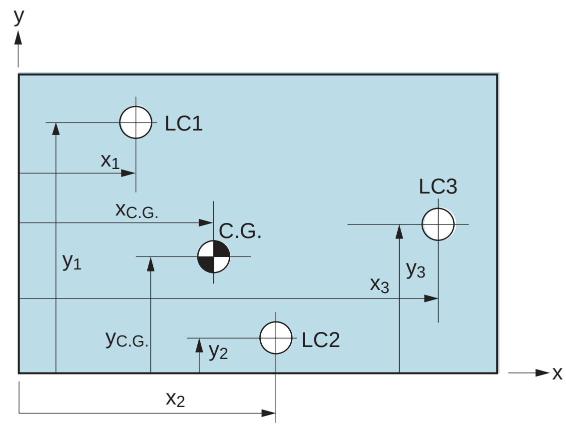


Figure 29 Setup for Experimental Finding of the C.G. Location

Hz. When the machine is installed on five linear isolators with rubber flexible elements selected in accordance with the manufacturer's recommendations, different for different mounting points (line 2, f_n = 15 Hz), the maximum amplitude of the relative vibrations (resulting in waviness of the ground surface) was 0.35 μm. However, when the grinder was installed on five indentical CNF isolators with rubber flexible elements (line 1, f_n = 20 Hz, or about two times stiffer than the linear isolators), the maximum relative vibration amplitudes was 0.25 μm, about 30% lower.

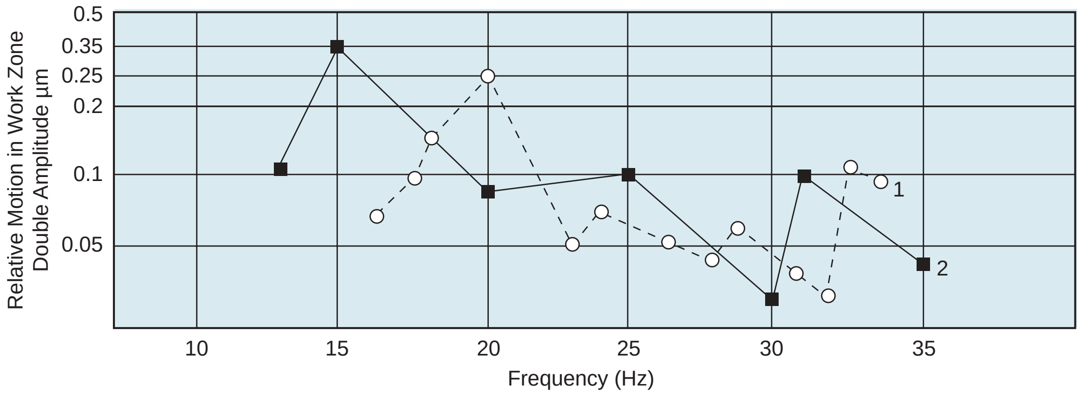


Figure 33 Amplitude of Relative Motion in Work Zone with: 1 - Regular (Linear) Isolators; 2 - CNF Isolators

7.0 CONNECTIONS OF SPRING ELEMENTS

7.1 Springs in Parallel

These combine like electrical resistance in series. This is the case when several springs support a single load, as shown in Figure 34. The springs are equivalent to a single spring, the spring constant of which is equal to the sum of the spring constants of the constituent springs. The spring constant k of the single equivalent spring is given by:

$$k = k_1 + k_1 + k_1. \quad (27)$$

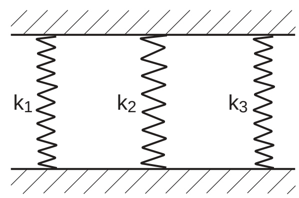


Figure 34 Parallel Connection of Springs

7.2 Springs in Series

The series connected springs in Figure 35 combine like electrical resistances in parallel. The equivalent single spring is softer than any of the component springs. The spring constant k of the equivalent single spring is given by:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}. \quad (28)$$

If n springs are in series, this formula is readily extended to:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ... + \frac{1}{k_n}. \quad (29)$$

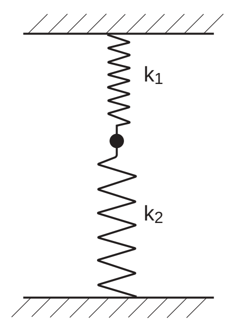


Figure 35 Series Connection of Springs

7.3 Spring Connected Partly in Parallel and Partly in Series

Obtain equivalent spring constants for each set of parallel or series springs separately and then combine. For example, in Figure 36, the springs k_1 and k_2 are equivalent to a single spring, the spring constant of which, k_{e1}, is given by:

$$\frac{1}{k_{e1}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2} \quad \text{or} \quad k_{e1} = \frac{k_1 k_2}{k_1 + k_2} \quad (30a)$$

The three springs, k_3, k_4, k_5 in parallel, are equivalent to a single spring, the spring constant of which, k_{e2}, is given by:

$$k_{e2} = k_3 + k_4 + k_5 \quad (30b)$$

Now equivalent springs k_{e1} and k_{e2} are in series. Hence, the spring constant k of the equivalent spring for the entire system is:

$$\frac{1}{k} = \frac{1}{k_{e1}} + \frac{1}{k_{e2}} \quad \text{or} \quad k = \frac{(k_1 k_2)(k_3 + k_4 + k_5)}{k_1 k_2 + (k_1 + k_2)(k_3 + k_4 + k_5)} \quad (30c)$$

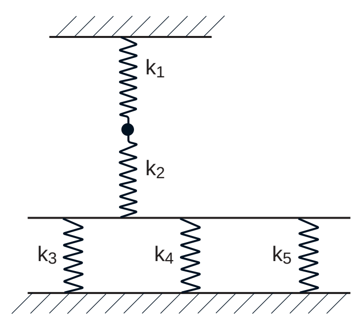


Figure 36 Mixed Connection of Springs

8.0 3-D OBJECT DRIVEN BY VIBRATORY FORCE AND TORQUES

Figure 37 shows an object with its C.G. at C, mounted on 4 flexible mounts and acted upon by a disturbing harmonic force F_y in the y-direction (vertical) and/or by torques, T_x, T_y and T_z acting singly or in combination about the x, y and z axes, which are principal inertia axes passing through the C.G. (point C).

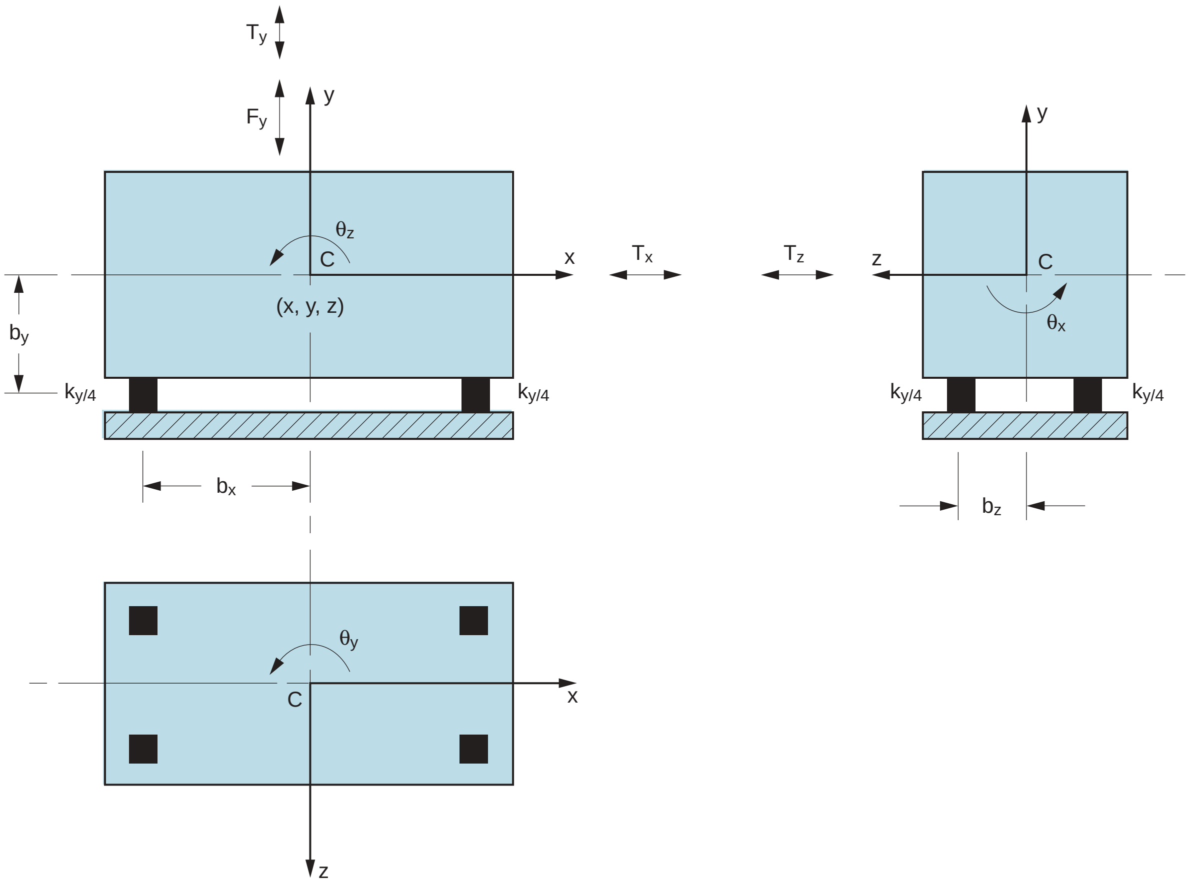


Figure 37 Solid Body on Vibration Isolators

The four mounts are symmetrically disposed relative to the C.G., their location defined by distances b_x, b_y and b_z from the axes, as shown. The mass moments of inertia about the principal inertia axes are I_x, I_y and I_z, respectively. As a result of the external force and torques, the object motion is (a) a displacement of C.G., maximum values of which are denoted by translational motions of the C.G. (x, y, z) and (b) rotations of the object (from equilibrium) about the coordinate axes (θ_x, θ_y, θ_z). These displacements are generally small relative to the major dimensions of the object.

Let: M = mass of object (W/g where W is weight of the object, g = 386 in/sec^2 = 9.8 m/sec^2);
k_y = total vertical stiffness of the four supports in lb./in. or N/m; i.e., 4 times the stiffness of each support if all four supports are identical
k_s = total horizontal or shear stiffness of the four supports; i.e., 4 times the horizontal stiffness of each support, if all supports are identical and for each support $k_x = k_z = k_s$, lb./in. or N/m;
ω = angular frequency of sinusoidally applied force and torques (rad/sec)

Damping is assumed to be negligible.

8.1 Displacement of the Object

Due to F_y only:

$$y = \frac{F_y}{k_y - M\omega^2} \qquad (31)$$

Due to T_z only:

$$x = \frac{T_z b_y k_s}{I_z M\omega^4 - \omega^2 (I_z k_s + k_y b_x^2 M + k_s b_y^2 M) + k_y k_s b_x^2} \qquad (32)$$

$$\theta_z = \frac{T_z (k_s - M\omega^2)}{I_z M\omega^4 - \omega^2 (I_z k_s + k_y b_x^2 M + k_s b_y^2 M) + k_y k_s b_x^2} \qquad (33)$$

Due to T_x only: $$z = \frac{T_x b_y k_s}{I_x M\omega^4 - \omega^2 (I_x k_s + M k_y b_z^2 + M b_y^2 k_s) + k_y k_s b_z^2} \quad (34)$$

$$\theta_x = \frac{T_x (k_s - M\omega^2)}{I_x M\omega^4 - \omega^2 (I_x k_s + M k_y b_z^2 + M b_y^2 k_s) + k_y k_s b_z^2} \quad (35)$$

Due to T_y only: $$\theta_y = \frac{T_y}{k_s (b_x^2 + b_z^2) - I_y \omega^2} \quad (36)$$

In these equations F_y, T_x, T_y and T_z represent peak values of the corresponding applied force or torques.

8.2 Undamped Natural Frequencies

Source	Mode	Equation	
F_y	Translation along y-axis	$\omega_1 = \sqrt{\frac{k_y}{M}}$	(37)
T_z	Rotation about axes parallel to z-axis	$\omega_2 = \sqrt{A - \sqrt{A^2 - \frac{k_y k_s b_x^2}{I_z M}}}$	(38)
		$\omega_3 = \sqrt{A + \sqrt{A^2 - \frac{k_y k_s b_x^2}{I_z M}}}$	(39)
		where $A = \frac{k_s}{2M} + \frac{k_y b_x^2 + k_s b_y^2}{2I_z}$	(40)
T_x	Rotation about axes parallel to x-axis	$\omega_4 = \sqrt{B - \sqrt{B^2 - \frac{k_y k_s b_z^2}{I_x M}}}$	(41)
		$\omega_5 = \sqrt{B + \sqrt{B^2 - \frac{k_y k_s b_z^2}{I_x M}}}$	(42)
		where $B = \frac{k_s}{2M} + \frac{k_y b_z^2 + b_y^2 k_s}{2I_x}$	(43)
T_y	Rotation about y-axis	$\omega_6 = \sqrt{\frac{k_s (b_x^2 + b_z^2)}{I_y}}$	(44)

8.3 Mount Deflections

If the object motions in all coordinates are as expressed in 8.1 (x, y, z, θ_x, θ_y, θ_z,) and if the coordinates of the mounting point (vibration isolators) are (X, Y, Z) in the equilibrium position, then their deflections (ΔX, ΔY, ΔZ) from equilibrium due to the applied force/torques are:

$$\begin{aligned} \Delta X &= x - \theta_z Y + \theta_y Z \\ \Delta Y &= y - \theta_x Z + \theta_z X \\ \Delta Z &= z - \theta_y X + \theta_x Y \end{aligned} \quad (45)$$

provided the deflections are small relative to the object dimensions.

However, if the effects of more than one disturbing force/torque are to be combined, the corresponding deflections of each mount must be combined *vectorially*, not be added algebraically, as in Equation (45).

General Comments

1. It is desirable to make sure that the disturbing forces and torques operate at frequencies sufficiently far removed from the computed natural frequencies, so that resonance conditions are avoided.
2. The compliance of the vibration mounts in compression and shear should be such that their combined compliance yields natural frequencies which are sufficiently lower than the frequencies of the disturbing forces and torques (hopefully at least by a factor of 2.5).

3. The displacements (max. deflections) of the mounts can be calculated from Equation (45) for any given single disturbing force or torque. If several force/torques act simultaneously, vector addition of forces in different directions is required, and Equation (45) cannot be used.
4. The case of a horizontal disturbing force has not been considered in this presentation.
5. Other things being equal, the best arrangement for the mounts is to arrange them so that their resultant force passes through the center of gravity of the equipment and that its line of action is a principal axis. If there is a resultant torque about the center of gravity, its direction should be about a principal axis through the center of gravity. However, if this arrangement is impractical, it need not be adhered to.

9.0 COMPLEX DRIVING FORCES

When the disturbing forces are neither sinusoidal nor suddenly applied, the vibration analysis becomes more complicated. While it is more difficult to give general guidelines or methods of analysis, one can consider every force-time variation as composed of components of different frequencies, each being a multiple of the basic (usually driving) frequency. Mathematically, this is known as expanding an arbitrary function into a Fourier series. Once these frequency components (harmonics) are determined, each one being sinusoidal at a different frequency, any component can be analyzed like a sinusoidal force. This can provide at least some understanding of the vibration phenomenon. Often the lowest-frequency (fundamental) component predominates and is the most important component to analyze. It is possible, however, that the design of the vibration isolation system will appear unfeasible on the basis of an analysis of only the fundamental component, whereas the exact analysis would show that a vibration isolation mounting can be useful; i.e., sometimes an analysis of components of several frequencies may be required [1]. This, however, may be quite difficult. In such cases, resolving an arbitrary force-time variation into several harmonics can provide some insight.

The following represents data in the Fourier series (decomposition into several harmonics) of some representative force-time variations in Figure 38, which are neither sinusoidal nor sudden. Each force is assumed to be a periodic function of the time;

$\lambda = \tau/T$, where τ is pulse width, T is the process period;
ω = fundamental frequency.

The Fourier expansions for these forcing functions are given in Table 2.

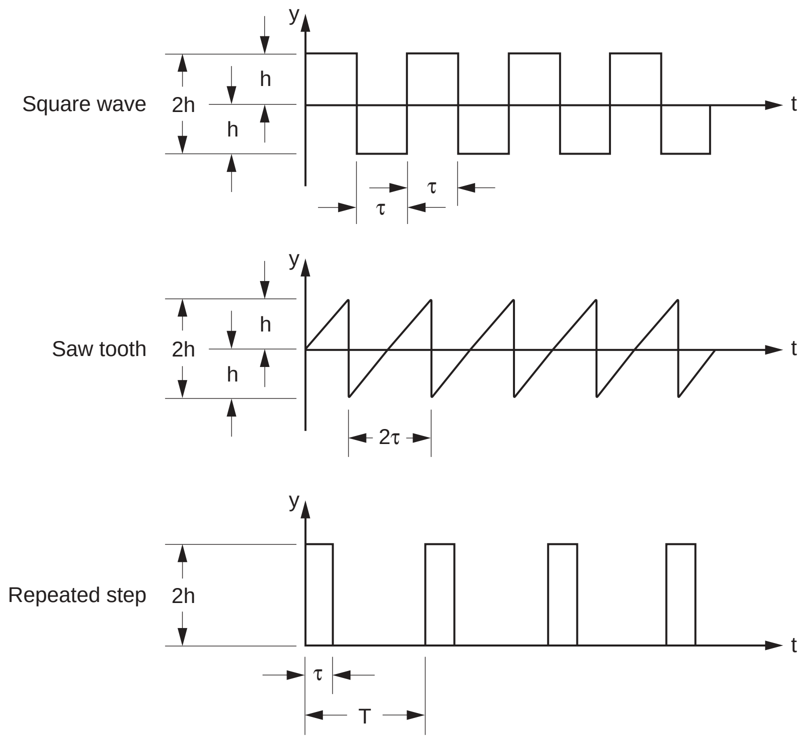


Figure 38 Typical Periodic Nonsinusoidal Vibratory Processes

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5. Other things being equal, the best arrangement for the mounts is to arrange them so that their resultant force passes through the center of gravity of the equipment and that its line of action is a principal axis. If there is a resultant torque about the center of gravity, its direction should be about a principal axis through the center of gravity. However, if this arrangement is impractical, it need not be adhered to.

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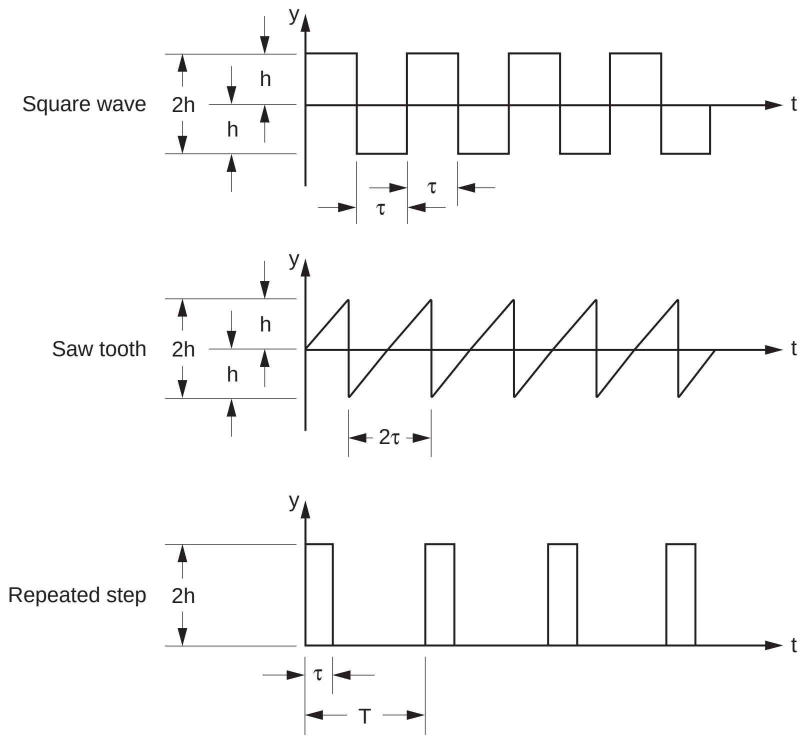

Figure 38 Typical Periodic Nonsinusoidal Vibratory Processes

TABLE 2 FOURIER EXPANSIONS FOR VIBRATORY PROCESSES IN FIGURE 38 (angles in radians)

Wave Shape Function	Harmonic Amplitude as Fractions of 2h (ω = fundamental frequency)					
Frequency of Harmonics	ω	2ω	3ω	4ω	5ω	6ω
Square wave	$\frac{2}{\pi}$	0	$\frac{2}{3\pi}$	0	$\frac{2}{5\pi}$	0
Saw tooth	$\frac{1}{\pi}$	$\frac{1}{2\pi}$	$\frac{1}{3\pi}$	$\frac{1}{4\pi}$	$\frac{1}{5\pi}$	$\frac{1}{6\pi}$
Repeated steps	$\frac{2\sin \pi\lambda}{\pi}$	$\frac{2\sin 2\pi\lambda}{2\pi}$	$\frac{2\sin 3\pi\lambda}{3\pi}$	$\frac{2\sin 4\pi\lambda}{4\pi}$	$\frac{2\sin 5\pi\lambda}{5\pi}$	$\frac{2\sin 6\pi\lambda}{6\pi}$

To illustrate this approach in a particular case, let's consider a connecting-rod motion of a slider-crank mechanism, Figure 39, as in internal-combustion engines. This motion can be shown to have the following Fourier expansion:

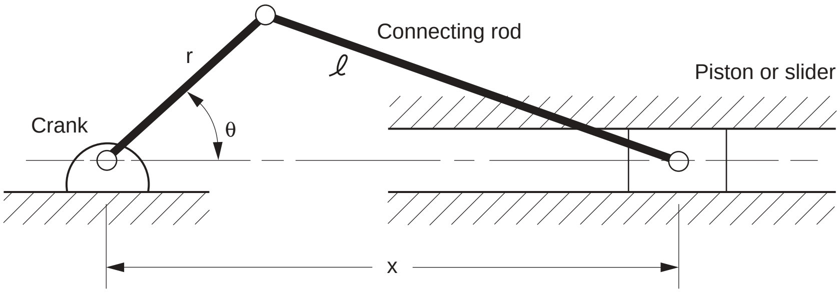

Figure 39 Schematic of a Slider-Crank Mechanism

r = crank length, in.
ℓ = connecting rod length, in.
θ = crank angle, rad or deg.
x = piston placement (piston motion in-line with crank pivot), in.
ω = crank speed, assumed constant, rad/sec
a = piston acceleration, in/sec^2

$$\frac{x}{r} = A_0 + \cos\theta + \frac{1}{4} A_2 \cos 2\theta - \frac{1}{16} A_4 \cos 4\theta + \frac{1}{36} A_6 \cos 6\theta \ldots \quad (46)$$

$$-\frac{a}{r\omega^2} = \cos\theta + A_2 \cos 2\theta - A_4 \cos 4\theta + A_6 \cos 6\theta \ldots \quad (47)$$

where A_2, A_4, A_6 are given as follows in Table 3 [4].

TABLE 3 COEFFICIENTS FOR FOURIER EXPANSION OF CONNECTING ROD MOTION

ℓ/r	A_2	A_4	A_6
3.0	0.3431	0.0101	0.0003
3.5	0.2918	0.0062	0.0001
4.0	0.2540	0.0041	0.0001
4.5	0.2250	0.0028	—
5.0	0.2020	0.0021	—

10.0 DESIGN PROBLEM EXAMPLES

The following are a number of problems intended to familiarize the reader with the basic applications of vibration isolators. More advanced techniques which would result in stiffer isolators while achieving adequate isolation can be found in [1].

NOTE: In the following problems, unless otherwise stated, it is assumed that the loads are evenly distributed among the mounting points.

Problem No. 1

A metal tumbling unit weighing 200 lbs and driven by a 950 rpm motor is to be mounted for at least 81% vibration isolation effieciency from the tumbling drum and motor unbalance (one cycle per revolution, or 950 cpm) using 4 cylindrical mounts in shear. Select the isolators.

The weight load per mounting is (1/4) x 200 lbs = 50 lbs. From the basic vibration chart, Figure 12, a forcing frequency of 950 cpm (~ 16Hz) and 81% isolation lead to a point of intersection corresponding to a static deflection of 0.25 in.

Cylindrical mount Part Number V10Z 2-311C, loaded in shear, has a deflection 0.32 in. at 50 lbs. Since this deflection is in excess of 0.25 in., the isolation will be greater than the design minimum. From the basic chart in Figure 12, it is seen to be between 85-90%.

Problem No. 2

Consider the tumbling unit of Problem No. 1 and suppose the motor speed were increased to 2500 rpm. What isolators could be used, allowing loading both in shear and in compression?

From the basic vibration chart, Figure 12, for a forcing frequency of 2500 cpm and 81% isolation, we find a static deflection of about 0.037 in. Hence we must look for isolators with a load rating not less than 50 lbs and with a corresponding deflection of not less than 0.037 in. The following mounts can be considered:

Load in Compression	Load in Shear
V10Z 2-300C (0.078 in. deflection)	V10Z 2-330B (0.14 in. deflection)
V10Z 2-317C (0.078 in.)	V10Z 2-311C (0.31 in.)
V10Z 2-310B (0.138 in.)	
V10Z 2-314C (0.042 in.)	

Amongst these, the highest percentage of isolation is afforded by the mount with the largest deflection (V10Z 2-311C), provided that such a deflection is permissible.

Problem No. 3

A small business machine is to be mounted for 81% vibration isolation effiecency. The weight is 25 lbs and there are 4 mounting points. What additional information is required for the selection of the vibration isolation system?

Information which is needed is as follows: allowable vibration amplitudes of the machine, as a function of frequency; frequency of disturbing force; direction and point of application of disturbing force; space limitations, if any; ambient conditions, if unusual; mass and compliance distribution of machine – if not uniform.

Problem No. 4

A device contains 4 symmetrically located special-configuration isolators (Finger-Flex), Part Number V10R 4-1502D, each isolator deflecting just over 0.07 in. at 20 lb load. In order to obtain satisfactory vibration isolation, it is desired to increase the deflection from 0.07 in. to 0.14 in., the load remaining the same. How can this be done?

One way is to stack two (identical) mounts in series, see section 7.2, each of the four isolators being replaced by such a set.

Problem No. 5

A unit which is to be mounted for 81% vibration isolation efficiency has a forcing frequency of 1500 cpm (25 Hz), weighs 1080 lbs and is to use 6 vibration isolators in shear. Isolators with a female tap are required. Select an isolator model.

The load per isolator is 1080/6 = 180 lbs. At 1500 cpm and 81% isolation effieciency, the basic vibration chart, Figure 12, gives a static deflection of 0.10 in.

Isolator V10Z 2-308C loaded in shear has a deflection of about 0.13 in. at 180 lbs. This being in excess of 0.10 in., the degree of isolation is certainly satisfactory. This model has a female tap.

Problem No. 6

A 275 lb motor is mounted with cylindrical isolators V10Z 2-311C loaded in shear at six points, the forcing frequency being 1100 cpm (~ 18 Hz). What is the percentage of vibration isolation attained?

The load per isolator = 275/6 = 45.8 lbs, assuming mounts to be symmetrically located, so that load is evenly distributed. From the design information furnished in the catalog, the shear deflection of the isolator at this load is ~ 0.28 in.

From Figure 12, the point of intersection of 0.28 in. static deflection and forcing frequency of 1100 cpm gives an isolation efficiency of about 87%.

Problem No. 7

An air conditioner weighs 250 lbs and is driven by a motor at 1700 rpm. The unit is mounted in shear on four V10Z 2-317B cylindrical isolators. Is this design satisfactory?

The isolated unit is not properly installed because the maximum load rating for this isolator, as indicated in the catalog, is 21 lbs in shear and 40 lbs in compression. The load per mount is 250/4 = 62.5 lbs. Even if the isolator is installed so that it is loaded in compression, it would not be satisfactory, since the load (62.5 lbs) is significantly in excess of the 40 lbs recommended limit.

Mounts, which have sufficient load capacity, are as follows (with static deflection indicated):

Part Number	Static Deflection
V10Z 2-310B (Compression)	(0.175 in.)
V10Z 2-311C (Shear – marginal)	(0.38 in.)
V10Z 2-330B (Shear)	(0.175 in.)

The choice of isolators depends (amongst other matters) on the degree of isolation desired. With any of the above isolators, this will be in excess of 81% for the forcing frequency equal to motor rpm.

Problem No. 8

If, in the preceding problem, the air conditioner weighs 350 lbs, what is the choice of mounts?

The load/mount is 350/4 = 87.5 lbs. The following mounts can be considered (with static deflection indicated):

Part Number	Static Deflection
V10Z 2-314C (Compression)	(0.075 in.)
V10Z 2-311D (Compression)	(0.094 in.)
V10Z 2-330B (Shear)	(0.26 in.)

At 1750 cpm, 81% vibration isolation corresponds to a static deflection of 0.074 in.

Problem No. 9

A computer weighs 200 lbs. It is to be vibration isolated with 4 mounts. The forcing frequency is 1750 cpm (~ 29 Hz). If the isolators are to be loaded in compression, what models are available and what is the percentage of vibration isolation attained in each case?

The load per mount is 200/4 = 50 lbs. Hence, isolators with a load capacity of at least 50 lbs in compression are needed. For each isolator, the catalog contains data (table or plots) from which static deflection under a 50 lb load can be found. From the basic vibration chart, Figure 12, with this value of static deflection and a forcing frequency of 1750 cpm, the point of intersection defines the attained vibration isolation efficiency. Thus, the following isolators can be selected:

Type of Mount	Part Number	Static deflection, in., at 50 lb compression	Isolation Efficiency, %
Cylindrical	V10Z 2-317C	0.078 in.	82%
Cylindrical	V10Z 2-300C	0.078 in.	82%
Cylindrical	V10Z 2-310B	0.138 in.	91%
Special (Finger-Flex)	V10R 4-1506B	0.14 in.	~ 91%
Special (Finger-Flex)	V10R 4-1506C	0.09 in.	~ 85%

Problem No. 10

A 4-cylinder engine weighing 370 lbs and operating at 2800 rpm is to be isolated for 81% vibration isolation for one-per-revolution excitation frequency. Discuss the possible selection of isolators.

The lowest frequency to be isolated is 2800 cpm (~ 46.5 Hz). In general, it is desirable to arrange the mounts so that the resultant of the loads, supported by the mounts, passes through the C.G. This is the same condition (but stated differently) as the one described in Section 5.0 above. If the isolators are symmetrically arranged, and each isolator carries the same load, this usually means that the symmetry axis of the isolators passes through the C.G. In this case, we are concerned not only with the translational displacement of the engine as a whole, but also with engine rotation. In addition, flexible gas lines and the throttle linkage can vibrate and their vibration isolation may pose an additional problem.

At 2800 cpm and 81% isolation efficiency, the basic vibration chart, Figure 12, gives a static deflection of about 0.03 in. The load is 370/4 = 92.5 lbs per mount.

Consider rectangular mount V10Z 6-500B loaded in shear. This has a deflection of about 0.12 in. in shear, which can accommodate the rotation of the engine about the torque-roll axis. The mount deflection in compression would serve to accommodate the shock load in translation.

Problem No. 11

An 80 lb fan is to be vibration isolated in shear at four points with at least 93% vibration isolation efficiency when the fan is turning at 2000 rpm. Specify the mounts.

The main source of vibration is rotor unbalance, and the transmission to ground of the vertical component of this force, (which is sinusoidal) is undesirable (see [1] for isolation of other vibration components). Hence, consider Section 3.1, Equation (8), with negligible damping.

Solution #1:

From the basic vibration chart, Figure 12, 93% isolation at a forced frequency of 2000 cycles/minute (~ 33.3 Hz) corresponds to a static deflection of about 0.14 in. and to a natural frequency of about 500 cpm (~ 8.5 Hz).

Consider cylindrical vibration isolators, Part Number V10Z 2-300C loaded in shear, which deflects about 0.17 in. at 20 lbs and appears to be suitable for this application.

Solution #2: (Analytical)

When the isolation efficiency is 93%, the force transmissibility, μ_F is 1 – 0.93 = 0.07 or 7%. With zero damping (δ = 0), Equation (8) gives for $\delta = 0$:

$$\mu_F = \frac{1}{\pm\left(1 - \frac{f^2}{f_n^2}\right)} \tag{48}$$

where $\mu_F = 0.07$;

"+" is to be used when $f < f_n$, and

"–" is to be used when $f > f_n$.

Since for good isolation, $f > f_n$, "–" sign will be used.

Solving for f/f_n from Equation (48), we obtain $f/f_n = 3.91$.

Since f = 33.3 Hz, f_n = 8.52 Hz.

From Equation (4), solving for x_{st} = 0.344 cm = 0.136 in.

These calculations agree adequately with the values found from the chart in Figure 12.

Problem No. 12

Data as in problem 11, but damping is estimated at $c/c_{cr} = 0.1$, see Section 2.4 above. How would it change the specifications?

The force transmissibility, μ_F, corresponding to 93% vibration isolation efficiency, is 0.07 and the forcing frequency is 2000 cpm (33.3 Hz). From Figure 10, for damping ratio $c/c_{cr} = 0.1$ at $\mu = 0.07$, the frequency f/f_n = ~ 5.

Hence, f_n = 2000/5 = 400 cpm (~ 6.7 Hz).

From the basic chart, Figure 12, this natural frequency corresponds to a static deflection of ~ 0.21 in. Since the load remains at 20 lbs per mount, the isolators specified for Problem 11 are too stiff. Isolator V10Z 2-310B loaded in shear appears to be satisfactory (deflection ~ 0.33 in. at 20 lbs).

This problem could also have been solved by a computer program, or analytically. In the latter case, Equation (8) can be solved for f_n at the value c/c_{cr} = 0.1, f = 33.3 Hz.

Comparison of Problems 11 and 12 shows that viscous damping in isolators results in increasing transmissibility at the isolation frequency range (which starts from $f/f_n = \sqrt{2} = 1.41$); i.e., reducing effectiveness of isolation and requiring softer isolators to get the desired efficiency. This is the price to pay for very desirable reduction of resonance amplitudes. When the damping is not viscous but material damping, such as in isolators with rubber flexible elements, the deterioration of the high frequency isolation is minimal.

Problem No. 13 A Vibroactive Object (Machine)

A small machine tool weighs between 3.5 lbs and 5 lbs depending on the weight of the work piece. When the forcing frequency, which is generated by the vibration source inside the machine, is between 60-90 Hz. and again when it is within 200-400 Hz range, the vibration is objectionable. Design a vibration mount for a 3-point support with vibration isolation efficiency of not less than 81%.

In the absence of more information, we may assume that isolators have zero damping. If we isolate for the lowest objectionable forced frequency (60 Hz), that would take care of all the troublesome regions.

From the basic vibration chart, Figure 12, an 81% isolation ratio at a forced frequency of 60 Hz corresponds to a static deflection of about 0.019 in. The weight supported by each mounting ranges from 3.5/3 lbs to 5/3 lbs, or from 1.17 to 1.67 lbs. The natural frequency is read off from the chart at about 23 Hz. Hence, the vibration mount specification is:

0.019 in. deflection

1.17 lbs to 1.67 lbs supported weight.

Square mount V10Z 1-321B loaded in compression is a possibility. Considering the special configuration (Finger-Flex) mounts, Part Number V10R 4-1500A can be selected. Its deflection at 1.17 lbs is only about 0.03 in. In view of its construction, the spring rate of this mount increases rapidly with deflection and the special configuration unit would be both more economical in the use of space and more effective in taking care of overloads, if this should arise.

Problem No. 14 A Vibration/Shock Sensitive Object

Sensitive radio equipment is to be mounted with a 3-point suspension on a boat. Protection from engine disturbance is required, as well as from impacts of waves and from bumping against pier. The equipment weighs 54 lbs and the engine runs at 2000 rpm.

Here we have both steady vibrations at 2000 cycles/min as well as shock loads, caused by wave pounding and by bumping against the dock. We have no precise information on the latter and need to do the best we can.

For the steady vibration, consider Equation (8) with zero damping which becomes Equation (48). At 81% efficiency and forcing frequency of 2000 cycles/min, the basic vibration chart, Figure 12, gives a static deflection of about 0.058 in. The load per mount is 54/3 = 18 lbs. The natural frequency obtained from the chart is about 760 cycles/min = 12.7 Hz.

V10R 4-1504B ring-style special-configuration (Finger-Flex) mount approximately fulfills this condition.

In order to limit the effect of shock loads, conical bumpers may be added to limit the horizontal shock load, possibly with the V10Z 7-1020C type.

It can, however, also be made an arbitrary guess and assumption that the pier and waves effects are equivalent approximately to a 0.5 mph sudden change of horizontal velocity of the boat and try to design the vibration mount for this condition. This will provide some insight into how much of a sudden velocity can be expected to be cushioned by vibration mounting. This corresponds to Section 3.3.1 of horizontal motion and negligible damping ($c/c_{cr} = 0$).

It is also important to know how much force the sensitive radio equipment can take without damage. Often such a force is expressed as a g-load; i.e., how many times its own weight the equipment can survive. For example, a 1/2 g-load means that the object can withstand a maximum force of (1/2) (54) = 27 lbs without damage. Usually, the allowable shock loads are determined by testing. Let's assume that the maximum safe load on the radio equipment is 1g or 54 lbs.

From Equation (15), Section 3.3.1, we have

$$\frac{a_{max}}{g} = \frac{2\pi fV}{g} = 1,$$

where V = 0.5 mph = 8.8 in/sec = 0.22 m/sec. Hence, f = 7 Hz.

This frequency is quite low, and associated with undesirably large deflections of vibration isolators. This suggests using a cylindrical mount loaded in compression for the vertical (engine) vibrations and having reasonably large compliance in the horizontal (shear) mode to take care of some of the shock, with a conical bumper to limit excessive horizontal deflections.

For example, cylindrical mount V10Z 2-300A has 0.075 in. deflection at 20 lbs compressive load, while in shear, the deflection at 16 lbs is about 0.32 in., or six times as much. This is an overload, but might still be considered due to the infrequent occurences of the shock load.

The natural frequency in the shear mode based on the 16 lb load is about 5.7 Hz, which is 20% lower than the 7 Hz specified above.

From Equation (18), $\frac{d_{max}}{d_{st}} = \frac{a_{max}}{g} = 1$, thus d_{max} = 0.32 in. Note that d_{max} is computed as if the weight were supported in shear.

This is too large a maximum deflection. A conical bumper should be used to limit the deflection by 0.20 in., say. Alternatively, a stronger and stiffer mount should be considered, for example, V10Z 2-300B, which deflects 0.26 in. at 18 lbs in shear. The isolation effectiveness in compression is reduced to about 65%; and while the isolation ratio in shear is also reduced, so is the corresponding maximum deflection. In addition, the conical bumpers should be added. The final choice of mounts is a matter of judgment.

Problem No. 15

A single-cylinder gasoline engine drives a one-cylinder air compressor with belt. Both units are bolted to a light-gage metal pan, which is welded to the top of an air-receiver tank, which is in turn mounted to a four-wheel steel-tired dolly. The whole unit vibrates and walks all over the floor. The engine weighs 100 lbs and turns at 3000 rpm. The compressor weighs 120 lbs and turns at 1200 rpm. The tank weighs 25 lbs and the dolly weighs 50 lbs. What can be done?

Possibly good rubber tires on the dolly and/or wheel suspension would help. If the tank is mounted to the dolly, total weight is:

$$W = 100 + 120 + 75 = 295 \text{ lbs.}$$

The lowest-frequency disturbing force is that due to the air compressor; i.e., 1200 cycles/min = 20 Hz. At 81% vibration isolation efficiency, Figure 12 gives a static deflection of the isolator of about 0.15 in. Considering a 4-mount suspension, the load per mount is 74 lbs.

Cylindrical mount V10Z 2-310B would be a possibility, loaded in compression. If the dolly continues to move, since it weighs only 50 lbs, it might require a little softer material than the 40-durometer rubber, in order to effect more isolation.

Next, consider mounting on isolators the pan that holds the engine and air-compressor unit. The total weight here is 100 + 220 lbs and with the same static deflection of 0.15 in., a V10Z 2-310A mount would suffice in compression, considering the fact that the chart shows the V10Z 2-310B mounts to deflect 0.12 in. at 55 lbs. The lower-durometer mount (Type A, at 30-durometer) should, therefore, approximate the 0.15 in. required deflection. Note that the last letter in the mount identification specifies the approximate durometer hardness of the rubber (A = 30, B = 40, C = 50).

Problem No. 16   Isolation of a Punch Press (also see [1]).

This is one of the most difficult applications for isolation. Shock absorption is all that can be expected. Unit weighs 1500 lbs, sits on four feet, operates at 50-100 rpm, and is driven by a 5 H.P., 1750 rpm electric motor, the flywheel turning at 250 rpm.

While many vibration problems deal with sinusoidal or nearly sinusoidal forces and some (such as in package cushioning) deal with essentially sudden velocity changes, here we have a suddenly applied force, which is periodic, but not harmonic. The force-time variation is essentially that of the "Repeated Step" in Section 9.0.

If we assume that the punching operation of the press occurs, say, during 30° of crank rotation, then the λ in this case (Repeated Step, Section 9.0) is 30/360 = 1/12 = 0.08333. From Section 9.0, we find that the amplitude of the fundamental harmonic is $(2/\pi) \sin \pi\lambda$ or 0.164. This is only about 16% of the amplitude of the force pulse, and its frequency is operating frequency (50-100 rpm or 0.85 - 1.7 Hz).

Consider, however, the 4th harmonic (200-400 cycles/min). Its amplitude is $(2 \sin 4\pi\lambda)/4\pi$ = .1376 or 13.8%. This is not much less than the amplitude of the basic (fundamental) frequency. This shows that in the punch-press type of disturbing force, the higher harmonics cannot be neglected.

The fundamental frequency (50-100 cycles/min) is so low that isolation with vibration isolation mounts would lead to their excessive static deflections. However, it is conceivable that a practical vibration isolator would be successful in isolating some of the significant higher harmonics. For vibration isolation of punch presses, the following few rules might be useful (also see [1]).

1. Slow-speed presses should be mounted with mounts of greater deflection than high-speed presses.
2. Mount deflections used for presses by direct installation of vibration isolation mounts under their feet may vary from 1/32 in. to 3/4 in. depending largely on operating speed and stroke length, with the smaller deflection being the more common.
3. There may be several static deflections that will work, while other static deflections interspaced in between them will not work; i.e., 1/16 in. and 3/16 in. may work, while 1/8 in. may not work. This can be caused, at least in part, by the fact that a significant set of higher harmonics may be isolated at one deflection, but not at another.
4. Even the best mounting system will still transmit a significant amount of vibration and shock.
5. If the ultimate in isolation is required, the punch press must be attached solidly to an inertia block of large mass and the entire press and the block mounted on vibration isolators.

Problem No. 17

A relatively high-precision experiment is to be conducted in the laboratory of a textile plant. The laboratory floor vibrates at an amplitude of 0.0005 in. due to the operation of industrial sewing machines and other textile machinery. The basic floor-vibration frequencies are that excited by the industrial sewing machines, which operate in the 1500-5000 rpm range. It is desired to vibration isolate the test unit, which weighs 25 lbs, with a four-point mounting at not less than 81% isolation of displacement.

At 81% displacement isolation, the displacement transmissibility, μ_X is 0.19. It is calculated using the same equations (8) and (48) as for μ_F.

For zero damping, Equation (48) gives:

$$\mu_F = \frac{1}{\pm\left(1 - \frac{f^2}{f^2_n}\right)}$$

Taking f as the lowest sewing-machine speed (1500 cycles/min or 25 Hz) and μ_F = 0.19, we find f_n = 600 cycles/min = 10 Hz. The static deflection of the vibration isolators is determined from Equation (4), as x_{st} = 0.25 cm = 0.1 in. The same result can be obtained from Figure 12. The isolation specification, therefore, is 0.10 in. static deflection at a load of 25/4 = 6.25 lbs.

Considering cylindrical vibration isolators, mount V10Z 2-316B loaded in shear, has a 0.10 in. deflection at about 6.25 lbs. Soft mounts, such as this one, are often using shear deformation of the flexible elements.

Problem No. 18

Data as in Problem 17, except that system damping is estimated at 10% of critical (δ = ~ 0.63). Reevaluate the specification of the isolators.

In Problem 17, we found that the displacement transmissibility corresponding to 81% isolation is μ_x = 0.19; and that the lowest forcing frequency, f = 1500 cycles/min. = 10 Hz. From Figure 10, p.T1-11, which applies to μ_x as well as to μ_F, we find that the given value of the transmissibility at δ = ~ 0.63 yields a frequency ratio f/f_n = ~ 2.7. Hence, f_n = 1500/2.7 = ~ 9 Hz.

At this natural frequency, the basic vibration chart (Figure 12) gives a static deflection of about 0.117 in. The load per mount, as in Problem 17, is 6.25 lbs.

The isolator specification V10Z 2-316B of Problem 17 remains satisfactory.

Problem No. 19

An impact testing machine consists of a simple pendulum of length 4 feet and weight 5 lbs, which is initially horizontal. It is released and at the bottom of its swing impacts the test object. In this test, it comes to rest essentially instantaneously (inelastic impact). The object (equipment to be tested) weighs 100 lbs and is capable of withstanding accelerations up to 2g. Design a vibration isolation/mounting system so that the equipment will survive the impact test.

The velocity aquired by the pendulum in the 4 foot drop is

$V_o = \sqrt{2gh}$, = 193 in/sec (striking velocity), where g = 386 in/sec²; h = 4 ft. x 12 = 48 in.

The momentum of the pendulum just prior to impact is equal to the impulse "I" applied to the object. It is equal to the mass of the pendulum times its velocity,

$$I = \frac{5}{386} \times 193, \text{ or } 2.5 \text{ lb-sec.}$$

If the pendulum retains a residual velocity V_p' just after striking the test object, "I" would be computed from

$$I = (V_p - V_p') \times (\text{mass of pendulum}).$$

The impact result is an essentially sudden velocity change by V_1, of the equipment, which, can be calculated from Equation (20) as:

$$V = \frac{Ig}{W} \text{ in/sec.}$$

$$= \frac{(2.5)\,(386)}{100} \text{ in./sec.} = 9.65 \text{ in./sec.}$$

This value of V can be used in Equation (15), or

$$\frac{d_{max}}{d_{st}} = \frac{a_{max}}{g} = \frac{2\pi fV}{g}$$

with a_{max} = 2g and V = 9.65 in./sec. Then $f_n = g/\pi V$ = 1.35 Hz.

Realization of such low natural frequency (albeit, in a horizontal direction; less destabalizing than in the vertical direction) is a very special problem. It can be addressed by utilizing information in [1].

Problem No. 20 Vibration Isolation of High Precision Object

Formulate requirements for vibration isolation system (f_n and δ) for a projection aligner for semiconductor manufacturing for two conditions:

A - the apparatus is installed on the floor of a regular manufacturing plant so that for vertical direction $X_f(f)$ = const = 3.0 μm for frequencies 3 ~ 30 Hz and $X_f(f) = 3.0 \frac{30}{f}$ μm for frequencies f > 30 Hz; for the horizontal direction $X_f(f)$ = const = 2.5 μm for frequencies 2 ~ 20 Hz, and $X_f(f) = 2.5 \frac{20}{f}$ μm for frequencies f > 20 Hz.

B - floor vibration levels corresponding to line VC-B in Figure 15 (both for vertical and horizontal directions).

Vibration sensitivity of this apparatus to vertical and horizontal vibration of its frame (base) was experimentally determined and shown in Figure 40. These plots show what amplitude of vibration X_b at the given frequency results in a relative vibration amplitude in the working zone (image motion) not exceeding the tolerated amplitude Δ_o = 0.1 μm. The minima on these plots represent structural natural frequencies of the devices. At each frequency f, transmissibility from the base to the work zone is $\mu_f = \Delta_o/X_b$.

Since the vibration sensitivity μ_f of this precision object is known (can be easily calculated from the experimentally obtained plots in Figure 40) then Expression (12b) can be used for specifying vibration isolation parameters.

Table 4 gives the values of μ_f (Δ_o divided by the ordinate of the plot in Figure 40 for a given frequency) calculated for critical points from the plots in Figure 40 for vertical and horizontal directions, respectively.

Table 4 also contains values of Φ_{Av} and Φ_{Ah} calculated for these points using Equation (12b) and vertical and horizontal floor vibration amplitudes specified in A.

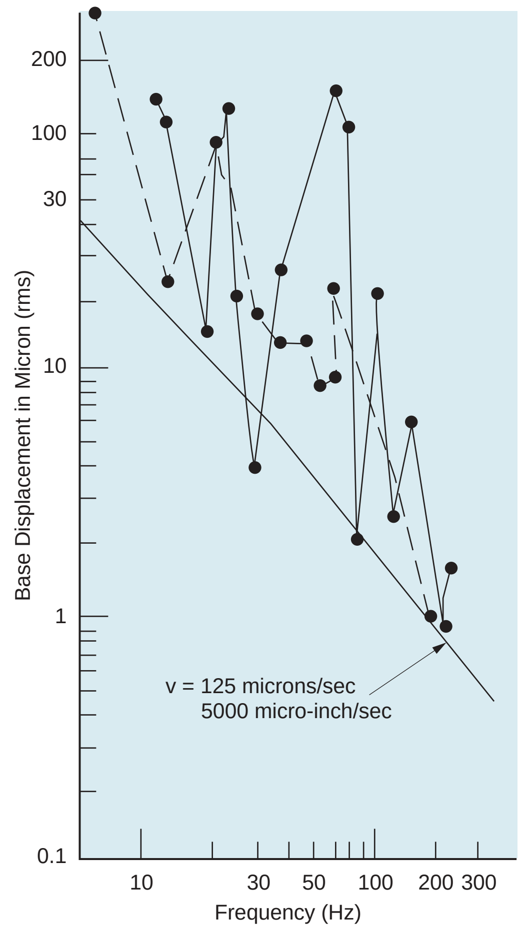

Figure 40 Vibration Sensitivity for Projection Aligner

Perkin-Elmer Microlign Mod. 341 for 0.1 μm Image Motion (Solid Line - Limit of Vertical Floor Vibration Amplitude, Broken Line - Limit of Horizontal Floor Vibration Amplitude).

TABLE 4 VIBRATION ISOLATION SYNTHESIS FOR FIGURE 40

A. Vertical Direction (Y-axis)			
f Hz	**μ (f)**	**Φ_{Av}** Hz	**Φ_{Bv}** Hz
11	0.0083	4.51	12.9
12	0.010	12.3	36.6
20	0.087	7.0	26.9
25	0.0091	26.9	116
30	0.056	13.0	61
32	0.303	6.3	29.7
41	0.05	22.5	106
70	0.0077	128	601

B. Horizontal Direction (X-axis)			
f Hz	**μ (f)**	**Φ_{Ah}** Hz	**Φ_{Bh}** Hz
7	0.0033	13.7	23.1
12	0.05	6.05	37.5
22	0.125	22.3	78
65	0.071	49.6	174
70	0.090	49.2	172
100	0.090	84	294

The values of Φ_{Bv} and Φ_{Bh} were calculated using floor vibration levels corresponding to line VC-B in Figure 15 (both for vertical and horizontal directions). Since plots in Figure 15 are given for vibratory velocity V_f, vibration displacement amplitudes X_f were calculated for each frequency of interest as $X_f = V_f/2\pi f$.

Values of Φ_A calculated per Specification A are interesting only for comparison, since high precision microelectronic production equipment is never used in conventional plant facilities, only in specially designed buildings complying with some of VC criteria.

It can be seen from Table 4A that the lowest value of Φ_{Av} (case A) for vertical direction is 4.51 Hz. If vibration isolators with medium damping $\delta_v = 0.6$ are used, then from Equation (12a) the required vertical natural frequency $f_v = 4.51\sqrt{0.6} = 3.04$ Hz. However, if isolators made of rubber with high damping $\delta_v = 1.2$ are used, then $f_v = 4.51\sqrt{1.2} = 5.0$ Hz, which can be realized by passive isolators with soft rubber flexible elements.

Much stiffer isolators ($f_{vz} > 14$ Hz) can be used to comply with values of Φ_{Bv}, per Specification B, which represent (according to not very stringent requirement VC-B) floor conditions at the microelectronics industry installations.

A similar situation is seen in Table 4B; however, realization of natural frequencies corresponding to Φ_{Bh} (4.7 Hz for $\delta_v = 0.6$, 6.63 Hz for $\delta_v = 1.2$) in horizontal directions with elastomeric isolators does not present any difficulty; even much lower values can be easily realized.

T1 Technical Section

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Appendix 1 – Useful Formulas in Vibration Analysis

www.vibrationmounts.com Phone: 516.328.3662 Fax: 516.328.3365

Natural Frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} = \frac{1}{2\pi}\sqrt{\frac{g}{x_{st}}}$$

where f_n = natural frequency in cycles-per-second (Hz)

k = spring constant (lbs/in, N/m)

m = mass of load (lb mass, kg mass)

g = gravitational constant, 386 in/sec^2 or 9.8 m/sec^2

W = weight of load, m•g (lb or N)

x_{st} = static deflectionof the spring (in or m)

$$f_n \approx \frac{3.13}{\sqrt{x_{st}}} \text{ cycles/sec} = \frac{188}{\sqrt{x_{st}}} \text{ cycles/min if } x_{st} \text{ is in inch}$$

$$\approx \frac{5}{\sqrt{x_{st}}} \text{ cycles/sec} = \frac{300}{\sqrt{x_{st}}} \text{ cycles/min if } x_{st} \text{ is in cm}$$

Damped Natural Frequency

$$f_{dn} = f_n\sqrt{1-\left(\frac{c}{c_{cr}}\right)^2} = f_n\sqrt{\frac{1-\delta^2}{4\pi^2}}$$

where δ = $2\pi\,(c/c_{cr})$ = log (A_n/A_{n-1}) logarithmic decrement

c = damping constant (lb-sec/in or N-sec/m)

c_{cr} = critical damping constant = $2\sqrt{km}$

A_n = n^{th} amplitude of vibration

Natural Frequency of Torsional Vibrations

$$f_t = \frac{1}{2\pi}\sqrt{\frac{k_t}{I}}$$

where k_t = torsional stiffness (lb-in/rad or N-m/rad)

I = polar mass moment of inertia (lb-in-sec^2 or kg-m^2)

(continued)

Transmissibility

$$\mu_F = \mu_x = \frac{m_f}{m + m_f}\sqrt{\frac{1 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}}$$

μ_F = force transmissibility

μ_x = motion transmissibility

m = mass of load

m_f = mass of base (foundation)

For $m_f = \infty$:

$\mu_F < 1$ for $f \geqslant 1.41\, f_n$

For $m_f = \infty$ and $\delta \approx o$ (negligible damping):

$$\mu_F = \frac{1}{\pm\left(1 - \frac{f^2}{f_n^2}\right)}$$

At resonance ($f/f_n = 1$), with some damping:

$$(\mu_F)_{max} = (\mu_x)_{max} \approx \frac{m_f}{m + m_f}\,\frac{\pi}{\delta}$$

PHYSICAL PROPERTIES OF FIVE STANDARD STRUCTURAL RUBBER COMPOUNDS

Compound Numbers**	R-325-BFK	R-430-BFK	R-530-BFK	R-630-BFK	R-725-BFK
Shear modules, lb per sq in.	50	70	95	140	195
Logarithmic decrement of amplitude* (referred to base 10)	.041	.055	.14	.23	.35
Successive amplitude ratio*	.91	.88	.72	.59	.45
Percent energy loss due to hysteresis, per cycle of vibration	17	22	47	65	80
Specific heat	.47	.43	.40	.38	.35
Thermal conductivity in B.T.U., per sq ft per hr for a temp gradient of 1°F per in. thickness	.97	1.04	1.08	1.15	1.26
Velocity of sound in rubber rods, ft per sec	115	165	210	345	750

* The logarithmic decrement given here represents the negative of the power to which 10 must be raised in order to obtain the ratio of any two consecutive amplitudes (on the same side of zero deflection) as unexcited vibration dies out. For instance, if the logarithmic decrement is 0.2, the ratio of one amplitude to the preceding one is

$$10^{-0.2} = \frac{1}{10^{0.2}} = \frac{1}{1.585} = 0.631 = \text{successive amplitude ratio.}$$

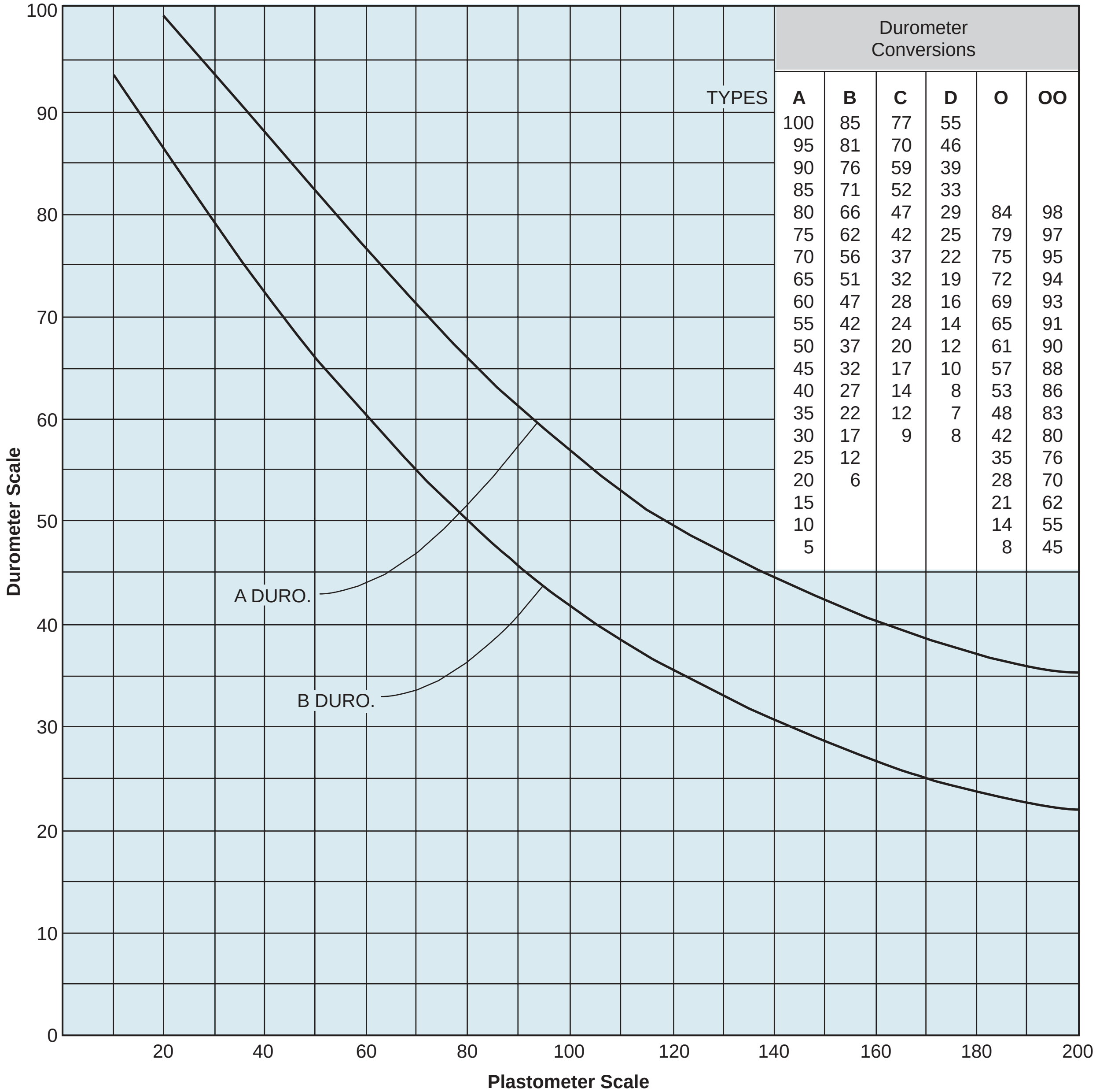
(Ordinarily, logarithmic decrement is referred to natural logarithm base e, and if such values are required, they would be 2.30 times the values given here.)

** Table from *U.S. Rubber Engineering Guide* #850 p. 25

COMPARATIVE PROPERTIES OF RUBBER AND RELATED MATERIALS

SAE Abbreviation	Butyl HR	Ethylene Propylene EPT	Hypalon CSM	Natural Rubber NR	Neoprene (Chloro-prene) CR	Nitrol (GR-A) NBR	Silicone SI	Styrene Butadiene (GR-S) SBR	Urethane PU	Flouro-Elastomer (Viton) HK
Cost Relative to Natural Rubber	110%	110%	150%	100%	110%	125%	850%	85%	450%	2000%
Tensile of Compounded Stocks	2000 psi	3000 psi	3000 psi	3500 psi	3000 psi	2500 psi	800 psi	2500 psi	8000 psi	2000 psi
Durometer	40-75	30-100	55-95	30-90	30-90	40-95	45-85	40-90	65-95	50-90
Elongation	fair	good	fair	excellent	excellent	good	fair	good	good	good
Aging	excellent	excellent	excellent	good	excellent	excellent	excellent	good	excellent	excellent
Heat Aging	excellent	excellent	good	good	very good	excellent	excellent	good	excellent	excellent
Sunlight Aging	good	excellent	excellent	poor	good	poor	good	poor	excellent	excellent
Lubricating Oil Resistance	poor	poor	good	poor	good	excellent	fair	poor	good	good
Aromatic Oil Resistance	poor	poor	poor	poor	fair	good	poor	poor	good	good
Animal-Vegetable Oils Resistance	excellent	poor	good	fair	excellent	good	good	fair	fair	good
Flame Resistance	poor	poor	excellent	poor	good	poor	fair	poor	poor	good
Tear Resistance	good	good	excellent	good	good	fair	poor	fair	excellent	fair
Abrasion Resistance	good	good	excellent	excellent	excellent	good	poor	good	excellent	fair
Compression Set Resistance	fair	fair	good	good	fair	good	fair	fair	excellent	good
Permeability to Gases	very low	good	good	fair	low	fair	fair	fair	good	excellent
Dielectric Strength	good	good	good	excellent	fair	poor	good	excellent	fair	good
Freedom from Odor	good	fair	excellent	excellent	good	fair	fair	fair	good	fair
Maximum Temperature (°F)	250	300	250	210	260	260	600	215	250	500
Minimum Temperature (°F)	-50	-50	-50	-65	-50	-60	-150	-60	-60	-40

FOR RUBBER AND PLASTICS
DUROMETER — PLASTOMETER CONVERSION CHART*



Durometer Conversions

TYPES A	B	C	D	O	OO
100	85	77	55		
95	81	70	46		
90	76	59	39		
85	71	52	33		
80	66	47	29	84	98
75	62	42	25	79	97
70	56	37	22	75	95
65	51	32	19	72	94
60	47	28	16	69	93
55	42	24	14	65	91
50	37	20	12	61	90
45	32	17	10	57	88
40	27	14	8	53	86
35	22	12	7	48	83
30	17	9	8	42	80
25	12			35	76
20	6			28	70
15				21	62
10				14	55
5				8	45

Conversions Are Approximate Values Dependent on Grades and Conditions of Materials Involved

*Courtesy of Shore Mfg. Co., New York

Durometer Hardness of Some Rubber Compounds		
Hardness (Shore A)	**ASTM Designation**	**Load Rating**
30	R-325-BFK	A
40	R-430-BFK	B
50	R-530-BFK	C
60	R-630-BFK	D
70	R-725-BFK	

Technical Section: Shaft Couplings

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1.0 INTRODUCTION

A coupling is a design component intended to connect shafts of two mechanical units, such as an electric motor and a hydraulic pump or compressor driven by this motor, etc. As stated in the Resolution of the First International Conference on Flexible Couplings [1, 3], "...a flexible coupling, although it is relatively small and cheap compared to the machines it connects, is a critical aspect of any shaft system and a good deal of attention must be paid to its choice at the design stage." The following is a brief engineering data on couplings. More details are available in [1, 3].

The application considerations for couplings are numerous. The most important are the following:

- Torque and Horsepower
- Allowable Shaft Misalignment
- Lateral and Axial Flexibility of Coupling
- Torsional Flexibility
- Backlash
- Rotational Velocity Error
- Service Conditions

2.0 APPLICATION CONSIDERATIONS

Flexible couplings are designed to accommodate various types of load conditions. No one type of coupling can provide the universal solution to all coupling problems; hence many designs are available, each possessing construction features to accommodate one or more types of application requirements. Successful coupling selection requires a clear understanding of application conditions. The major factors governing coupling selection are discussed below.

2.1 Torque and Horsepower

The strength of a coupling is defined as its ability to transmit a required torque load, frequently in combination with other factors.

Hence, a coupling may be selected whose rated torque capacity is many times greater than needed. For example, in a coupling subject to wear and increasing backlash, a useful torque rating would depend chiefly on backlash limitations rather than strength. For manually operated drives, the torque imposed through improper handling may be in excess of the drive torque required. Couplings are frequently specified in horsepower capacity at various speeds.

Horsepower is a function of torque and speed, and it can be readily determined from the formula:

$$HP = \frac{NT}{63{,}000}$$

where N = rotational speed in rpm and T= torque in lb. in. This relationship is graphically represented in Figure 1.

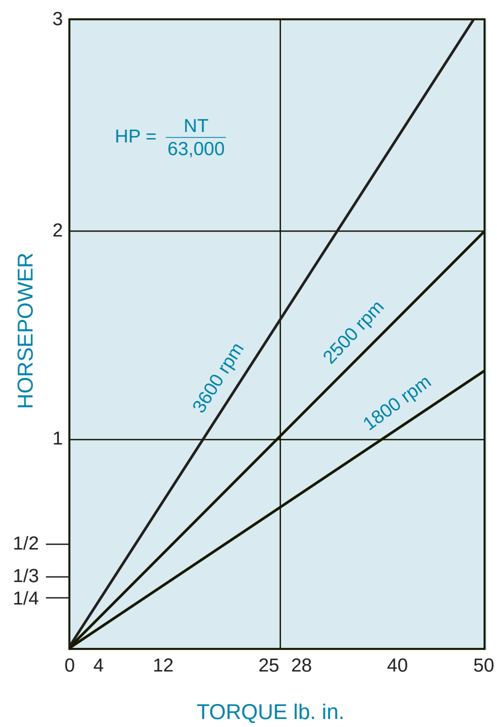


Figure 1 Relationship Between Horsepower, Torque and Rotational Speed

2.2 Shaft Misalignment

Shaft misalignment can be due to unavoidable tolerance build-ups in a mechanism or intentionally produced to fulfill a specific function. Various types of misalignment, as they are defined in AGMA Standard 510.02, are shown in Figure 2.

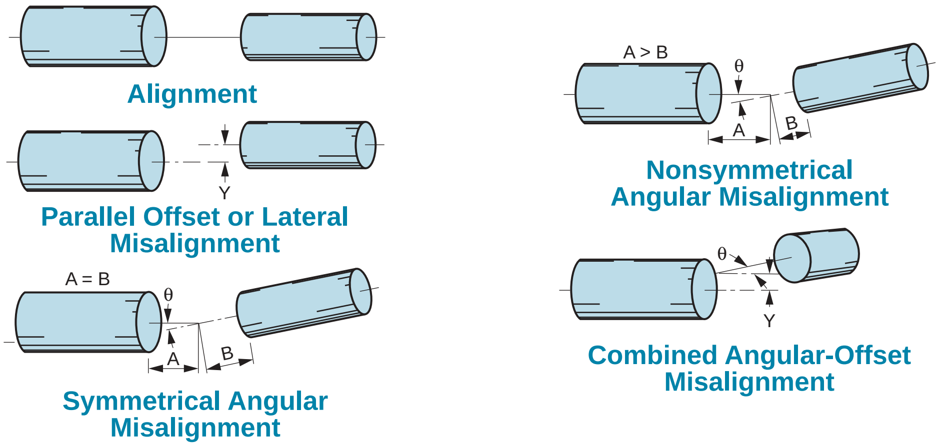


Figure 2 Various Types of Shaft Misalignment

2.3 Lateral and Axial Flexibility of Couplings

Lateral and axial flexibility of couplings are factors frequently overlooked. The term *flexible* does not mean that the coupling gives complete freedom of relative movement between the coupled shafts. More properly, flexible couplings give a limited freedom of the relative movement. Some forces are needed to make a flexible coupling flex. These forces are either lateral (at right angles to the shafts), or axial, or a combination of both. Lateral forces may produce a bending moment on the shafts and a radial load on the shaft support bearings. Axial force can produce undesirable thrust loads if not considered in the original design. Universal Cardan joints and Oldham couplings impose friction-generated lateral loads on the bearings. The elastomeric types of couplings will produce lateral forces in proportion to their stiffness. These issues are addressed below in Section 3.0.

2.4 Torsional Flexibility

Torsional flexibility of a coupling is the torsional (twisting) elastic deformation induced in a flexible coupling while transmitting torque. In some applications using encoders, it may be essential that the torsional flexibility be very low so as not to introduce reading errors caused by the angular displacements. On the other hand, torsional deflection may be desirable for reducing torque oscillations and peak torques in driving high inertia and/or dynamic loads.

2.5 Backlash

Backlash is the amount of rotational play inherent in flexible couplings which utilize moving parts. In some applications, this "slack" may not be objectionable, but in an application in servo-controlled systems, such as described in the previous paragraph, backlash would rule out couplings of this type.

2.6 Rotational Velocity Error

In addition to the types of error already described, universal joints produce an error because of their kinematic behavior. If the input speed into a single universal joint is held constant, then the output will produce cyclic fluctuations in direct relation to the operating angles of the input and output shafts. This will be described more fully in the section dealing with Universal Joints.

2.7 Service Conditions

Service conditions encompass factors such as temperature, operating medium, lubrication, accessibility for maintenance, etc., and should be reviewed before a final selection is made.

3.0 GENERAL CLASSIFICATION OF COUPLINGS AND THEIR PERFORMANCE CHARACTERISTICS

Couplings play various roles in machine transmissions. According to their role in transmissions, couplings can be divided into four classes:

1. *Rigid Couplings.* These couplings are used for rigid connection of precisely aligned shafts. Besides torque, they also transmit bending moment and shear force if any misalignment is present, as well as axial force. The three latter factors may cause substantial extra loading of the shaft bearings. The principal areas of application: long shafting; very tight space preventing use of misalignment-compensating or torsionally flexible couplings; inadequate durability and/or reliability of other types of couplings.

2. *Misalignment-Compensating Couplings.* Such couplings are required for connecting two members of a power-transmission or motion-transmission system that are not perfectly aligned. "Misalignment" means that components that are coaxial by design are not actually coaxial, due either to assembly errors or to deformations of subunits and/or their foundations, Figure 2. The latter factor is of substantial importance for transmission systems on nonrigid foundations.

If the misaligned shafts are rigidly connected, this leads to elastic deformations of the shafts, and thus to dynamic loads on bearings, vibrations, increased friction losses in power transmission systems, and unwanted friction forces in motion transmission, especially in control systems.

Misalignment-compensating couplings are used to reduce the effects of imperfect alignment by allowing nonrestricted or partially restricted motion between the connected shaft ends. Similar coupling designs are sometimes used to change bending natural frequencies/modes of long shafts.

When only misalignment compensation is required, rigidity in torsional direction is usually a positive factor, otherwise the dynamic characteristics of the transmission system might be distorted. To achieve high torsional rigidity together with high mobility/compliance in misalignment directions (radial or parallel offset, axial, angular), torsional and misalignment-compensating displacements in the coupling *have to be separated* by using an intermediate compensating member. Frequently, torsionally rigid "misalignment-compensating" couplings, such as gear couplings, are referred to in the trade literature as "flexible" couplings.

3. *Torsionally Flexible Couplings.* Such couplings are used to change the dynamic characteristics of a transmission system, such as natural frequency, damping and character/degree of nonlinearity. The change is desirable or necessary when severe torsional vibrations are likely to develop in the transmission system, leading to dynamic overloads in power-transmission systems.

Torsionally flexible couplings usually demonstrate high torsional compliance to enhance their influence on transmission dynamics.

4. *Combination Purpose Couplings* are required to possess both compensating ability and torsional flexibility. The majority of the commercially available connecting couplings belong to this group.

3.1 Rigid Couplings

Typical rigid couplings are shown in Figure 3. Usually, such a coupling comprises a sleeve fitting snugly on the connected shafts and positively connected with each shaft by pins, Figure 3a, or by keys, Figure 3b. Sometimes two sleeves are used, each positively attached to one of the shafts and connected between themselves using flanges, Figure 3c. Yet another popular embodiment is the design in Figure 3d wherein the sleeve is split longitudinally and "cradles" the connected shafts.

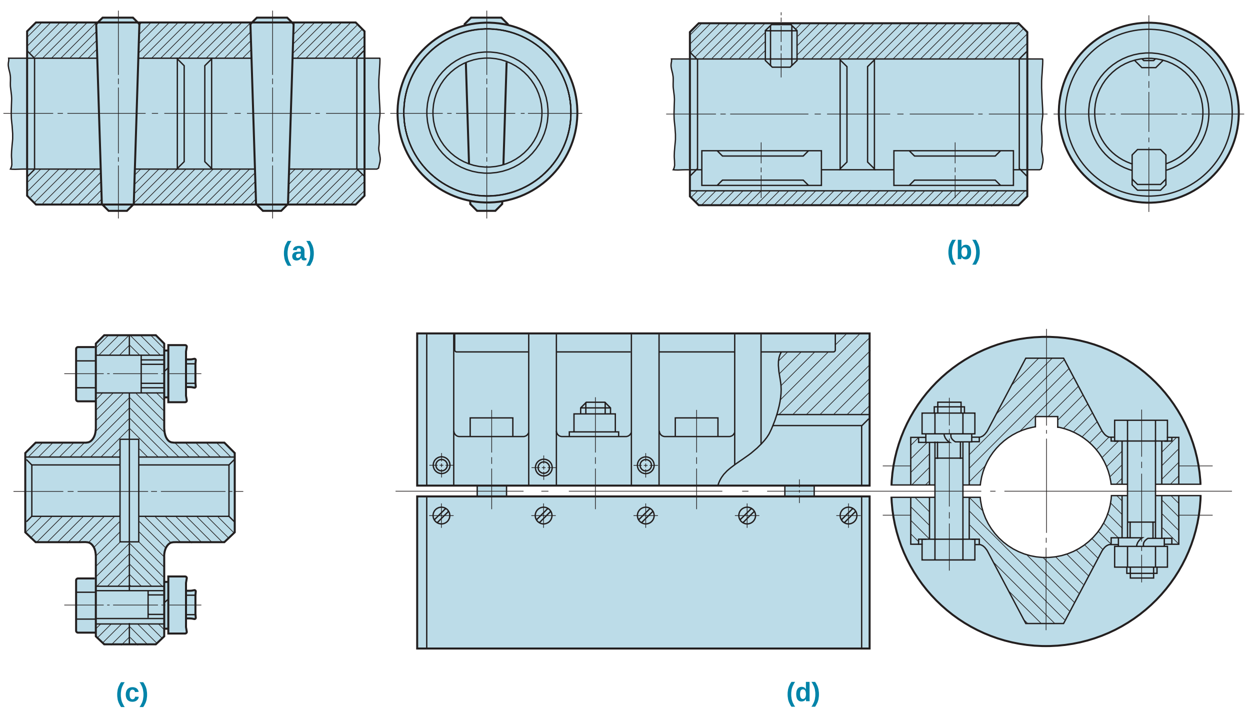


Figure 3 Examples of Rigid Couplings

3.2 Misalignment-Compensating Couplings

Misalignment-compensating couplings have to reduce forces caused by an imperfect alignment of connected rotating members (shafts). Since components which are designed to transmit higher payloads can usually tolerate higher misalignment-caused loads, a ratio between the load generated in the basic misalignment direction (radial or angular) to the payload (rated torque or tangential force) seems to be a natural design criterion for purely misalignment-compensating couplings.

All known designs of misalignment-compensating (torsionally rigid) couplings are characterized by the presence of an intermediate (floating) member located between the hubs attached to the shafts being connected. The floating member has mobility relative to both hubs. The compensating member can be solid or composed of several links. There are two basic design subclasses:

(2a) Couplings in which the displacements between the hubs and the compensating member have a frictional character (examples: Oldham coupling, Figure 4; universal Cardan Joint, Figure 5; gear coupling, Figure 6.)

(2b) Couplings in which the displacements are due to elastic deformations in special elastic connectors (e.g., "K" Type Flexible Coupling, Figure 7).

3.2.1 Selection Criterion for Frictional Misalignment-Compensating Couplings

For Subclass *(2a)* couplings designed for compensating the offset misalignment, the radial force F_{com} acting from one hub to another and caused by misalignment, is a friction force equal to the product of friction coefficient f and tangential force F_t at an effective radius R_{ef}, $F_t = T/R_{ef}$, where T is transmitted torque,

$$F_{com} = f F_t = \frac{f T}{R_{ef}} \qquad (1)$$

Since motions between the hubs and the compensating member are of a "stick-slip" character, with very short displacements alternating with stoppages and reversals, f might be assumed to be the static friction coefficient.

When the rated torque T_r is transmitted, then the selection criterion is

$$\frac{F_{com}}{T_r} = \frac{f}{R_{ef}} \qquad (2)$$

or the ratio representing the selection criterion does not depend on the amount of misalignment; lower friction and/or larger effective radius would lead to lower forces on bearings of the connected shafts.

Similar conclusion stands for couplings compensating angular misalignments (Cardan joints or universal, or simply, U-joints). While U-joints with rolling friction (usually, needle) bearings have low friction coefficient, f for U-joints with sliding friction can be significant if the lubrication system is not properly designed and maintained.

3.2.1a Oldham Couplings

Oldham couplings consist of three members. A floating member is trapped by 90° displaced grooves between the two outer members which connect to the drive shafts, as shown in Figure 4.

Oldham couplings can accommodate lateral shaft misalignments up to 10% of nominal shaft diameters and up to 3° angular misalignments.

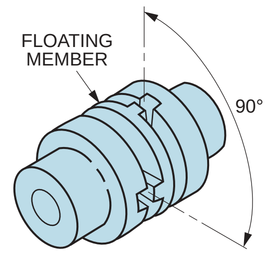


Figure 4 Oldham Coupling

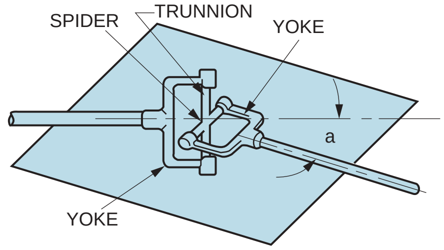


Figure 5 Universal Cardan Joint

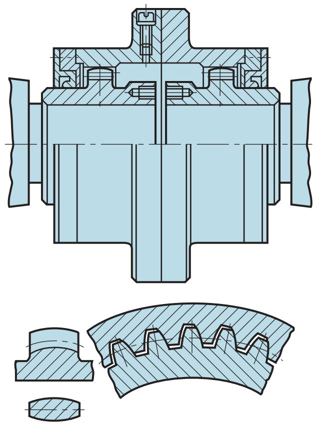


Figure 6 Gear Coupling

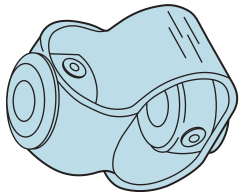


Figure 7 K-Type Elastomeric Coupling/Joint

T2 Technical Section

Lubrication is a problem but can, in most applications, be overcome by choosing a coupling that uses a wear-resistant plastic in place of steel or bronze floating members.

Some advantages of Oldham couplings:

- High torsional stiffness;
- No velocity variation as with universal joints;
- Substantial lateral misalignments possible;
- High torque capacity for a given size;
- Ease of disassembly.

Shortcomings of Oldham couplings:

- Limited angular misalignment of shafts;
- Need for lubrication due to relative sliding motion with stoppages, unless wear-resistant plastic is employed;
- Nylon coupling has reduced torque capacity;
- Significant backlash due to initial clearances for thermal expansion and inevitable wear;
- Are not suitable for small misalignments;
- Suitable only for relatively slow-speed transmissions;
- Possible loss of loose members during disassembly.

Oldham couplings with rubber-metal laminated bearings [1] have all the advantages of the generic Oldham couplings without their shortcomings.

3.2.1b Universal or U-joints [2]

3.2.1b.1 General

A universal joint, Figure 5, is a positive, mechanical connection between rotating shafts, which are not parallel, but intersecting. It is used to transmit motion, power, or both. It is also called the *Cardan joint* or *Hooke joint*. It consists of two *yokes*, one on each shaft, connected by a cross-shaped intermediate member called the *spider* having four *trunnions* providing for rotatable connections with the yokes. The angle between the two shafts is called the *operating angle*. It is generally, but not always, constant during operation. Good design practice calls for low operating angles, often less than 25°, depending on the application. Independent of this guideline, mechanical interference in the U-joint designs often limits the operating angle to a maximum (usually about 37.5°), depending on its proportions.

Typical applications of U-joints include aircraft, appliances, control mechanisms, electronics, instrumentation, medical and optical devices, ordnance, radio, sewing machines, textile machinery and tool drives.

U-joints are available with steel or plastic major components. Steel U-joints have maximum load-carrying capacity for a given size. U-joints with plastic body members are used in light industrial applications in which their self-lubricating feature, light weight, negligible backlash, corrosion resistance and capability for high-speed operation are significant advantages.

Recently developed U-joint designs with *rubber-metal laminated bearings* [1, 3] have even higher torque capacity and/or smaller sizes allowing for higher-speed operation, and can be preloaded without increasing friction losses, thus completely eliminating backlash. These designs do not require lubrication and sealing against contamination.

Constant velocity or *ball-jointed* universals are also available. These are used for high-speed operation and for carrying large torques. They are available in both miniature and standard sizes.

Motion transmitted through a U-joint becomes nonuniform. The angular velocity ratio between input and output shafts varies cyclically (two cycles per one revolution of the input shaft). This fluctuation, creating angular accelerations and increasing with the increasing angular misalignment, can be as much as ±15% at 30° misalignment. Effects of such fluctuations on static torque, inertia torque, and overall system performance should be kept in mind during the transmission design.

This nonuniformity can be eliminated (canceled) by using two connected in series and appropriately phased U-joints, Figure 8. While the output velocity becomes uniform, angular velocity fluctuation of the intermediate shaft cannot be avoided.

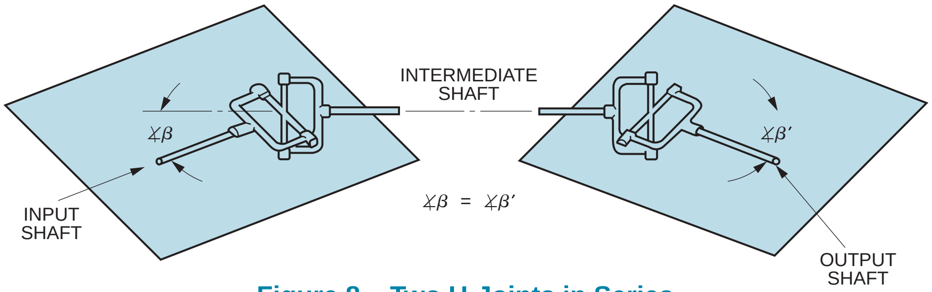


Figure 8 Two U-Joints in Series

Two U-joints in series can be used for coupling two laterally displaced (misaligned) shafts, while the single joint can only connect the angularly-misaligned shafts.

Advantages of a single U-joint:

- Low side thrust on bearings;
- Large angular misalignments are possible;
- High torsional stiffness;
- High torque capacity.

Shortcomings of a single U-joint:

- Velocity and acceleration fluctuations, especially for large misalignments;
- Lubrication is required to reduce friction and wear;
- Protection from contamination (sealing) is required;
- Shafts must be precisely located in one plane;
- Backlash is difficult to control;
- Static friction is increasing at very low misalignment (freezing), thus sometimes requiring an artificial misalignment in the assembly.

3.2.1b.2 Kinematics

Due to the velocity fluctuations, the angular displacements of the output shaft do not precisely follow those of the input shaft, but lead or lag, also with two cycles per revolution. The angular velocity variation is shown in Figure 9 for several operating (misalignment) angles β. The peak values of the displacement lead/lag, of input/output angular velocity ratio, and of angular acceleration ratio for different α are given in Table 1 [2]. As a qualitative guideline, for small β, up to ~10°, the deviations (errors) for maximum lead/lag angular displacements, for maximum deviations of angular velocity ratios from unity, and for maximum angular acceleration ratios are nearly proportional to the square of β.

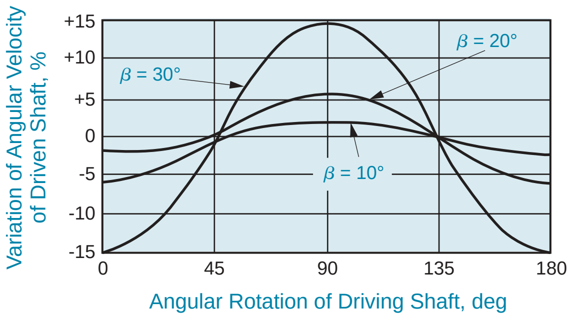


Figure 9 Angular Velocity Variations in U-Joint

The static torque transmitted by the output shaft is equal to the product of the input torque and the angular velocity ratio.

The angular acceleration generates inertia torque and vibrations. The total transmitted torque is a sum of inertia torque (the product of the angular acceleration and the mass moment of inertia of the output shaft and masses associated with it) and the nominal output torque.

The inertia torque often determines the ultimate speed limit of the joint. The recommended speed limits vary depending on β, on transmitted power, and on the nature of the transmission system. Recommended peak angular accelerations of the driven shaft vary from 300 rad/sec^2 to over 2000 rad/sec^2 in power drives. In light instrument drives, the allowable angular accelerations may be higher. For an accurate determination of the allowable speed, a stress analysis is necessary.

Example 1: Determining the Maximum Inertia Torque

A U-joint operates at 250 rpm with an operating angle β = 10°. Find the maximum angular displacement lead (or lag), maximum and minimum angular velocity of output shaft and maximum angular acceleration of output shaft.

If the system drives an inertial load so that the total inertial load seen by the output shaft (its own inertia and inertia of associated massive rotating bodies) can be represented by a steel circular disc attached to the output shaft (radius r = 3 in., thickness t = 1/4 in.), find the maximum inertia torque of the drive.

From Table 1 at β = 10°, the maximum displacement lead/lag = 0.439° = 26.3'. The maximum and minimum angular velocity ratios are given as 1.0154 and 0.9848, respectively. Hence, the corresponding output shaft speeds are:

$$\Omega_{max} = (250)(1.0154) = 254 \text{ rpm};$$

$$\Omega_{min}\ (250)(0.9848) = 246 \text{ rpm};$$

According to Table 1, the maximum angular acceleration ratio is

$$\alpha_{max}/\omega^2 = 0.0306 \text{ for } \beta = 10°.$$

$$\omega = [(250)\ (2\pi)]\ /\ (60) \text{ rad/sec} = 26.18 \text{ rad/sec}.$$

Hence, α_{max} = (0.0306)(26.18)^2 = 21.0 rad/sec^2. The weight, W, of the disc is given by W = π r^2 t γ, where γ denotes the density of steel and is equal to 0.283 lb/in^3.

TABLE 1 THE EFFECT OF SHAFT ANGLE (β) ON SINGLE UNIVERSAL JOINT PERFORMANCE FOR CONSTANT INPUT SPEED*

Operating Angle Between Shafts (β) Deg.	Maximum Load or Lag of Output Shaft Displacement (ε), Deg. Relative to Input Shaft Displacement	Maximum Angular Velocity Ratio (Ω_{max})	Minimum Angular Velocity Ratio (Ω_{min})	Maximum Angular Acceleration Ratio = $\frac{\alpha_{max}}{\omega^r}$, where α_{max} = Maximum Angular Acceleration of Output Shaft; ω = Angular Velocity of Input Shaft, rad/sec.
0	0.000	1.0000	1.0000	0.0000
1	0.004	1.0002	0.9998	0.0003
2	0.017	1.0006	0.9994	0.0012
3	0.039	1.0014	0.9986	0.0027
4	0.070	1.0024	0.9976	0.0049
5	0.109	1.0038	0.9962	0.0076
6	0.157	1.0055	0.9945	0.0110
7	0.214	1.0075	0.9925	0.0150
8	0.280	1.0098	0.9903	0.0196
9	0.355	1.0125	0.9877	0.0248
10	0.439	1.0154	0.9848	0.0306
11	0.531	1.0187	0.9816	0.0371
12	0.633	1.0223	0.9781	0.0442
13	0.744	1.0263	0.9744	0.0520
14	0.864	1.0306	0.9703	0.0604
15	0.993	1.0353	0.9859	0.0694
16	1.132	1.0403	0.9613	0.0792
17	1.280	1.0457	0.9563	0.0896
18	1.437	1.0515	0.9511	0.1007
19	1.605	1.0576	0.9455	0.1125
20	1.782	1.0642	0.9397	0.1250
21	1.969	1.0711	0.9336	0.1382
22	2.165	1.0785	0.9272	0.1522
23	3.372	1.0864	0.9205	0.1670
24	2.590	1.0946	0.9135	0.1826
25	2.817	1.1034	0.9063	0.1990
26	3.055	1.1126	0.8988	0.2162
27	3.304	1.1223	0.8910	0.2344
28	3.564	1.1326	0.8829	0.2535
29	3.835	1.1434	0.8746	0.2735
30	4.117	1.1547	0.8660	0.2946
31	4.411	1.1666	0.8572	0.3167
32	4.716	1.1792	0.8480	0.3400
33	5.034	1.1924	0.8387	0.3644
34	5.363	1.2062	0.8290	0.3902
35	5.705	1.2208	0.8192	0.4172
36	6.060	1.2361	0.8090	0.4457
37	6.428	1.2521	0.7986	0.4758
38	6.809	1.2690	0.7880	0.5074
39	7.204	1.2868	0.7771	0.5409
40	7.613	1.3054	0.7660	0.5762

*Reproduced with the permission of Design News from "The Analytical Design of Universal Joints" by S.J. Baranyi, Design News, Sept. 1, 1969

$W = \pi\,(3)^2\,(0.25)\,(0.283) = 2$ lb.

Inertia torque = $I\alpha_{max}$, where I = polar mass moment of inertia of disc (lb. in. sec^2),

$I = Wr^2 / 2g$,

where g = gravitational constant = 386 in/sec^2.

Hence, $I = [(2)\,(3)^2] / [(2)(386)] = 0.0233$ lb. in. sec^2.

Inertia torque = (21.0) (0.0233) = 0.489 lb. in. This inertia torque is a momentary maximum. The inertia torque fluctuates cyclically at two cycles per shaft revolution, oscillating between plus and minus 0.489 lb. in.

When system vibrations and resonances are important, it may be required to determine the harmonic content (Fourier series development) of the output shaft displacement as a function of the displacement of the input shaft. The amplitude of the mth harmonic (m > 1) vanishes for odd values of m, while for even values of m it is equal to (2/m) (tan 1/2β)^m, where β denotes the operating angle.

3.2.1b.3 Joint Selection (Torque Rating)

The torque capacity of the universal joint is a function of speed, operating angle and service conditions. Table 2 shows use factors based on speed and operating angle for two service conditions: *intermittent* operation (say, operation for less than 15 minutes, usually governed by necessity for heat dissipation) and *continuous* operation.

TABLE 2 USE FACTORS FOR THE TORQUE RATING OF UNIVERSAL JOINTS

Speed rpm	Intermittant Running Conditions								
	Angle of Operation - Degrees								
	0	3	5	7	10	15	20	25	30
1800	9	20	34	45	—	—	—	—	—
1500	8	16	28	39	—	—	—	—	—
1200	7	13	22	32	40	—	—	—	—
900	6	11	16	23	34	—	—	—	—
600	5	8	11	15	22	34	40	—	—
300	4	5	7	8	11	16	22	28	34
100	3	4	4	5	6	8	9	11	12

Speed rpm	Continuous Running Conditions								
	Angle of Operation - Degrees								
	0	3	5	7	10	15	20	25	30
1800	18	40	68	90	—	—	—	—	—
1500	16	32	55	78	—	—	—	—	—
1200	14	26	44	64	80	—	—	—	—
900	12	21	32	46	68	—	—	—	—
600	10	15	22	30	44	68	80	—	—
300	8	10	14	16	22	32	44	55	68
100	6	7	8	10	12	15	18	22	24

The torque capacity of a single Cardan joint of standard steel construction is determined as follows:

i. From the required speed (rpm), operating angle in degrees, and service condition (intermittent or continuous), find the corresponding use factor from Table 2.

ii. Multiply the required torque, which is to be transmitted by the input shaft, by the use factor. If the application involves a significant amount of shock loading, multiply by an additional dynamic factor of 2. The result must be less than the static breaking torque of the joint.

iii. Refer to the torque capacity column in the product catalog and select a suitable joint having a torque capacity not less than the figure computed in (ii) above.

If a significant amount of power is to be transmitted and/or the speed is high, it is desirable to keep the shaft operating angle below 15°. For manual operation, operating angles up to 30° may be permissible.

Example 2: Universal Joint Selection for Continuous Operation

A single universal joint is to transmit a continuously acting torque of 15 lb. in., while operating at an angle of 15° and at a speed of 600 rpm. Select a suitable joint.

From Table 2 for continuous operation, the use factor is given as 68. Note that there are blank spaces in the Table. If the combination of operating angle and speed results in a blank entry in the Table, this combination should be avoided. The required torque is (68) (15) = 1020 lb. in. There is no shock load and the dynamic factor of 2 does not apply in this case.

From the SDP/SI catalog, it is seen that there are two joints meeting this specification: A 5Q 8-D500 and A 5Q 8-D516, both with a torque capacity of 1176 lb. in. The first has a solid-shaft construction and the second a bored construction. The choice depends on the application.

Example 3: Universal Joint Selection for Intermittent Operation with Shock Loading

A single universal joint is to transmit 1/8 horsepower at 300 rpm at an operating angle of 15°. Select a suitable joint for intermittent operation with shock loading.

Here we make use of the equation:

$$\text{Torque} = \text{Horsepower} \times 63{,}025/300 \text{ lb. in.}$$

Hence, operating torque = (0.125)(63,025)/300 = 26.3 lb. in. From Table 2, for intermittent loads (300 rpm, 15°), the use factor is 16. Due to shock loading, there should be an additional dynamic factor of 2. Therefore, the rated torque = (26.3) (16) (2) = 842 lb. in. Thus, the same joints found in the previous example are usable in this case.

Example 4: Determining the Maximum Speed of an Input Shaft

A universal joint is rated at 250 lb. in., and operates at an angle of 12°, driving a rotating mass, which can be represented (together with the inertia of the driven shaft) by a steel, circular disc, radius r = 6", thickness t = 1/2", attached to the driven shaft. How fast can the input shaft turn if the inertia torque is not to exceed 50% of rated torque?

From Table 1, for $\beta = 12°$, we have $\alpha_{max}/\omega^2 = 0.0442$. The weight, W, of the disc is $W = \pi r^2 t \gamma$, where γ denotes the density of steel which is 0.283 lb. in^3.

Thus W = π (6)^2(0.5) (0.283) = 16 lb. The polar mass moment of inertia, I, of the disc is given by

$I = Wr^2 / 2g = (16)(6)^2 / (2)(386) = 0.746$ lb. in. sec^2.

The inertia torque = $I\alpha_{max}$ = 50% of 250 lb. in. = 125 lb. in. Since $I\alpha_{max} = (\alpha_{max} / \omega^2) \bullet (\omega^2 I) = (0.0442)(0.746)\ \omega^2 = 125$, $\omega^2 = 125 / 0.03297 = 3790.96$ or ω = 61.6 rad/sec = (61.6)(60) / 2π = 588 rpm.

Hence, if the inertia torque is not to exceed its limit, the maximum speed of the input shaft is 588 rpm. For joints made with thermoplastic material, consult the SDP/SI catalog, which contains design charts for the torque rating of such joints.

3.2.1b.4 Secondary Couples

In designing support bearings for the shafts of a U-joint and in determining vibrational characteristics of the driven system, it is useful to keep in mind the so-called *secondary couples* or *rocking torques*, which occur in universal joints. These are rocking couples in the planes of the yokes, which tend to bend the two shafts and rock them about their bearings. The bearings are thus cyclically loaded at the rate of two cycles per shaft revolution. The maximum values of the rocking torques are as follows:

Maximum rocking torque on input shaft = $T_{in}\tan\beta$;
Maximum rocking torque on output shaft = $T_{in}\sin\beta$,

where T_{in} denotes the torque transmitted by the input shaft and β the operating angle. These couples are always 180° out of phase. The bearing force induced by these couples is equal to magnitude of the rocking couple divided by the distance between shaft bearings.

For example, if the input torque, T_{in} is 1000 lb. in. and the operating angle is 20°, while the distance between support bearings on each shaft is 6 in., the maximum secondary couple acting on the input shaft is (1000) (tan 20°) = 364 lb. in. and on the output shaft it is (1000) (sin 20°) = 342 lb. in. The radial bearing load on each bearing of the input shaft is 364/6 = 60.7 lb. and it is 342/6 = 57 lb. for each bearing of the output shaft. The bearings should be selected accordingly.

It has been observed also that due to the double frequency of these torques, the critical speeds associated with universal drives may be reduced by up to 50% of the value calculated by the standard formulas for the critical speeds of rotating shafts. The exact percentage is a complex function of system design and operating conditions.

3.2.1b.5 Joints in Series

As mentioned in paragraph 3.2.1b.1, universal joints can be used in series in order to eliminate velocity fluctuations, to connect offset (nonintersecting) shafts, or both. Figure 8 shows a schematic of such an arrangement.

In order to obtain a constant angular-velocity ratio (1:1) between input and output shafts, a proper phasing of the joints is required. This phasing can be described as follows: two Cardan joints in series will transmit a constant angular velocity ratio (1:1) between two intersecting or nonintersecting shafts (see Figure 8), provided that the angle between the connected shafts and the intermediate shaft are equal ($\beta = \beta'$), and that when yoke 1 lies in the plane of the input and intermediate shafts, yoke 2 lies in the plane of the intermediate shaft and the output shaft.

If shafts 1 and 3 intersect, yokes 1 and 2 are coplanar.

When the above phasing has been realized, torsional and inertial excitation is reduced to minimum. However, inertia excitation will inevitably remain in the intermediate shaft, because this shaft has the angular acceleration of the output shaft of a single U-joint (the first of the two joints in series). It is for this reason that guidelines exist limiting the maximum angular accelerations of the intermediate shaft. Depending on the application, values between 300 rad/sec^2 and values in excess of 1000 rad/sec^2 have been advocated. In light industrial drives, the allowable speed may be higher. For an accurate determination of allowable speed, a stress analysis is necessary.

T2 Technical Section

Example 5: Determining the Maximum Speed of an Input Shaft in a Series

In a drive consisting ot two universal joints in series, phased so as to produce a constant (1:1) angular velocity ratio between input and output shafts, the angle between the intermediate shaft and input (and output) shaft is 20°. If the maximum angular acceleration of the intermediate shaft is not to exceed 1000 rad/sec^2, what is the upper limit of the speed of the input shaft?

From Table 1, with β = 20°, we find $\alpha_{max}/\omega^2 = 0.1250$.

Since α_{max} = 1000 rad/sec^2,

$\omega^2 = (\alpha_{max}) / (0.1250) = (1000) / 0.1250 = 8000$ rad/sec^2.

Hence, $\omega = \sqrt{8000}$ = 89.4 rad/sec = (89.4)(60) / 2π = 854 rpm.

Hence, the speed of the input shaft should not exceed 854 rpm. When the joint angle is less than or equal to 10°, Figure 10 can be used to compute the maximum speed or the maximum angular acceleration for a given input speed.

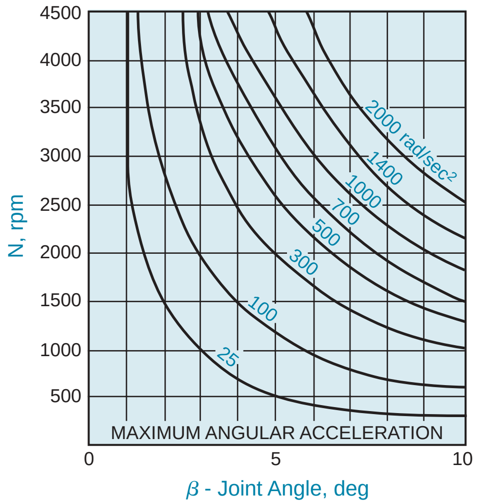


Figure 10 Maximum Angular Acceleration (rad/sec^2) of Output Shaft of Single U-Joint vs. Input Speed (rpm) and Operating Angle (degrees)

Example 6

Same as problem 5, except operating angle is 10°. Here we can use Figure 10. The intersection of $\beta = 10°$ and the 1000 rad/sec^2 curve yields N ≈ 1800 rpm. Hence, the speed of the input shaft should not exceed 1800 rpm. A more exact calculation, as in Example 5, yields N = 1726 rpm. For practical purposes, however, the value obtained from Figure 10 is entirely satisfactory.

3.2.2 Selection Criterion for Misalignment-Compensating Couplings with Elastic Connectors

For this class of couplings, assuming linearity of the elastic connectors,

$$F_{com} = k_{com} e, \qquad (3)$$

where e is misalignment value, k_{com} = combined stiffness of the elastic connectors in the direction of compensation. In this case,

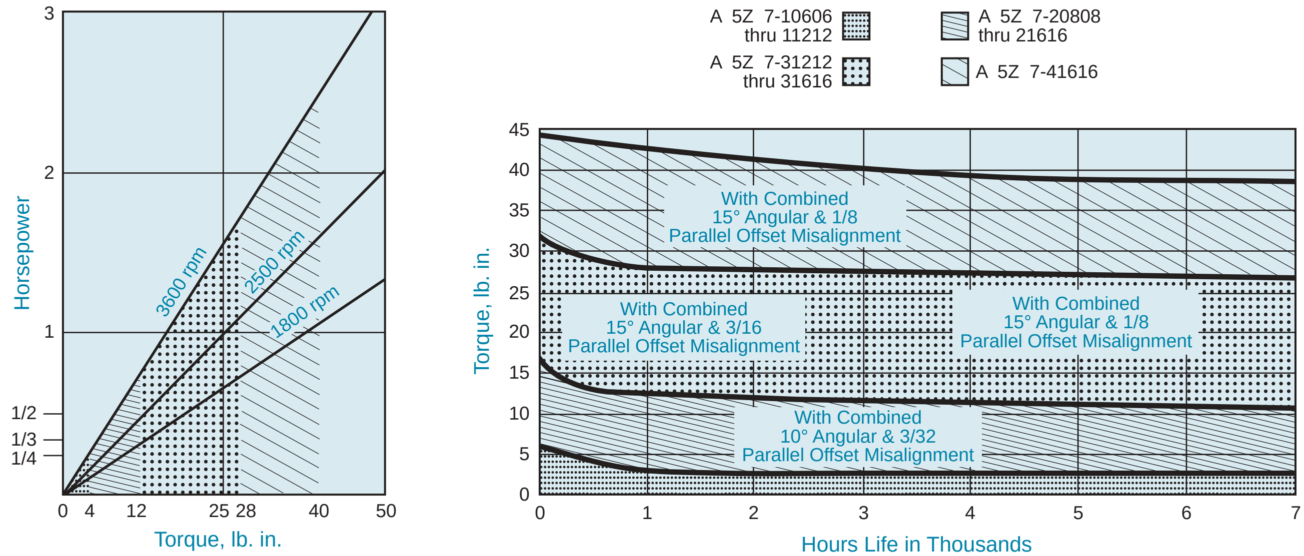
$$\frac{F_{com}}{T_r} = \frac{k_{com}}{T_r} e. \qquad (4)$$

Unlike couplings from Subclass (*2a*), Subclass (*2b*) (see p. T2-5) couplings develop the same radial force for a given misalignment regardless of transmitted torque, thus they are more effective for larger T_r. Of course, lower stiffness of the elastic connectors would lead to lower radial forces.

3.2.2.1 Designs of Elastic Misalignment-Compensating Couplings

Designs of Oldham couplings and U-joints with elastic connectors using high-performance thin-layered rubber-metal laminates are described in [1, 3].

K-Type flexible coupling, Figure 7, is kinematically similar to both Oldham coupling and to U-joint. By substituting an elastomeric member in place of the conventional spider and yoke of U-joint or the floating member of Oldham coupling, in construction such as in the design shown in Figure 7, backlash is eliminated. Lubrication is no longer a consideration because there are no moving parts and a fairly large amount of lateral misalignment can be accommodated. The illustrated coupling is available in the product section of this catalog. Please refer to Figure 11 for specific design data for four sizes of this type coupling. Figure 11b indicates that this coupling has high durability even with a combination of large lateral (offset) and large angular misalignments.



(a) Rated Horsepower/ Torque for Various rpm

(b) Service Life as a Function of Angular and Offset Misalignments for K-Type Couplings

Figure 11

3.3 Torsionally Flexible Couplings and Combination Purpose Couplings [3]

These two classes of couplings are usually represented by the same designs. However, in some cases only torsional properties are required, in other cases both torsional and compensation properties are important and, most frequently, these coupling designs are used as the cheapest available and users cannot determine what is important for their applications. Accordingly, it is of interest to look at what design parameters are important for various applications.

3.3.1 Torsionally Flexible Couplings

Torsionally flexible couplings are used in transmission systems when there is a danger of developing resonance conditions and/or transient dynamic overloads. Their influence on transmission dynamics can be due to one or more of the following factors:

Reduction of Torsional Stiffness and, Consequently, Shift of Natural Frequencies.

If resonance condition occurs before installation (or change) of the coupling, then shifting of the natural frequency can eliminate resonance; thus dynamic loads and torsional vibrations will be substantially reduced.

Increasing Effective Damping Capacity of a Transmission by Using Coupling Material with High Internal Damping or Special Dampers.

When the damping of a system is increased without changing its torsional stiffness, the amplitudes of torsional vibrations are reduced at resonance and in the near-resonance zone. Increased damping is especially advisable when there is a wide frequency spectrum of disturbances acting on a drive; more specifically, for the drives of universal machines.

Introducing Nonlinearity into the Transmission System.

If the coupling has a nonlinear "torque-angular deformation" characteristic and its stiffness is much lower then stiffness of the transmission into which it is installed, then the whole transmission acquires a nonlinear torque-angular deflection characteristic. A nonlinear dynamic system becomes automatically detuned away from resonance at a fixed-frequency excitation, the more so the greater the relative change of the overall stiffness of the system on the torsional deflection equal to the vibration amplitude.

Introducing Additional Rotational Inertia in the Transmission System.

This is a secondary effect since couplings are not conventionally used as flywheels. However, when a large coupling is used, this effect has to be considered.

Realizing the above listed effects of a properly selected torsionally-flexible coupling requires a thorough dynamic analysis of the transmission system.

3.3.2 Combination Purpose Couplings

Combination purpose couplings do not have a special compensating (floating) member. As a result, compensation of misalignment is accomplished, at least partially, by the same mode(s) of deformation of the flexible element which are called forth by the transmitted payload.

The ratio of radial (compensating) stiffness k_{com} and torsional stiffness k_{tor} of a combination purpose flexible coupling can be represented as [1,3]

$$\frac{k_{com}}{k_{tor}} = \frac{A}{R_{ex}^2}, \qquad (5)$$

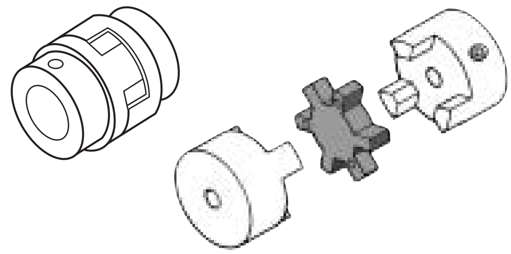
where R_{ex} is external radius of the coupling. The "Coupling Design Index" A (Figure 13f) allows one to select a coupling design better suited to a specific application. If the main purpose is to reduce misalignment-caused loading of the connected shafts and their bearings, for a given value of torsional stiffness, then the lowest value of A is the best, together with large external radius. If the main purpose is to modify the dynamic characteristics of the transmission, then minimization of k_{tor} is important.

Some combination purpose couplings are shown in Figure 12. The "modified spider" coupling (Figure 12b) is different from the conventional spider (jaw) coupling shown schematically in Figure 12a by four features: legs of the rubber spider are tapered, instead of staight; legs are made thicker even in the smallest cross section, at the expense of reduced thickness of bosses on the hubs; lips ℓ on the edges provide additional space for bulging of the rubber when legs are compressed, thus reducing stiffness; the spider is made from a very soft rubber. All these features lead to substantially reduced torsional and radial stiffnesses while retaining small size, which is characteristic for spider couplings.

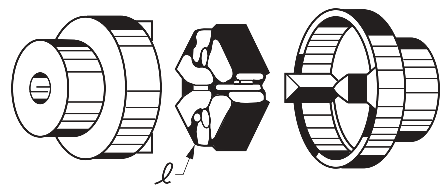
Plots in Figure 13 (a-d) give data for some widely used couplings on such basic parameters as torsional stiffness k_{tor}, radial stiffness k_{com}, external diameter D_{ex}, and flywheel moment WD^2 (W is weight of the coupling). Plots in Figure 13 (e-f) give derivative information: ratio k_{com} / k_{tor}, and design index A. All these parameters are plotted as functions of the rated torque.

Data for "toroid shell" couplings in Figure 13 are for the coupling as shown in Figure 12 (c) (there are many design modifications of toroid shell couplings). The "jaw coupling" for T = 7 Nm in Figure 13 (f) (lowest torque point on jaw coupling line) has a four-legged spider (z = 4), while all larger sizes have z = 6 or 8. This explains differences in A ($A \approx 1.9$ for z = 4, but A = 1.0 ~ 1.3 for z = 6,8). Values of A are quite consistent for a given type of coupling. The variations can be explained by differences in design proportions and rubber blends between the sizes.

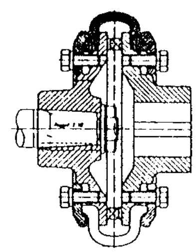
Using plots in Figure 13, one can more easily select a coupling type whose stiffnesses, inertia, and diameter are best suited for a particular application. These plots, however, do not address issues of damping and nonlinearity. Damping can be easily modified by the coupling manufacturer by a proper selection of the elastomer. As shown previously, high damping is very beneficial for transmission dynamics, and may even reduce thermal exposure of the coupling, as shown in [1,3]. More complex is the issue of nonlinear characteristics; a highly nonlinear (and very compact) coupling based on radial compression of cylindrical rubber elements is described in [1]. Couplings represented in Figure 13 are linear or only slightly nonlinear.



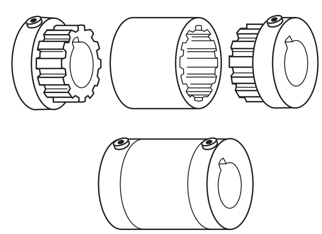
(a) Jaw (Spider) Coupling



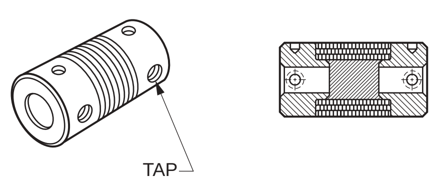
(b) Modified Spider Coupling (ℓ- lip providing bulging space for the rubber element)



(c) Toroid Shell Coupling



(d) Sleeve Coupling (Geargrip)



(e) Uniflex Coupling

Figure 12 Combination Purpose Couplings

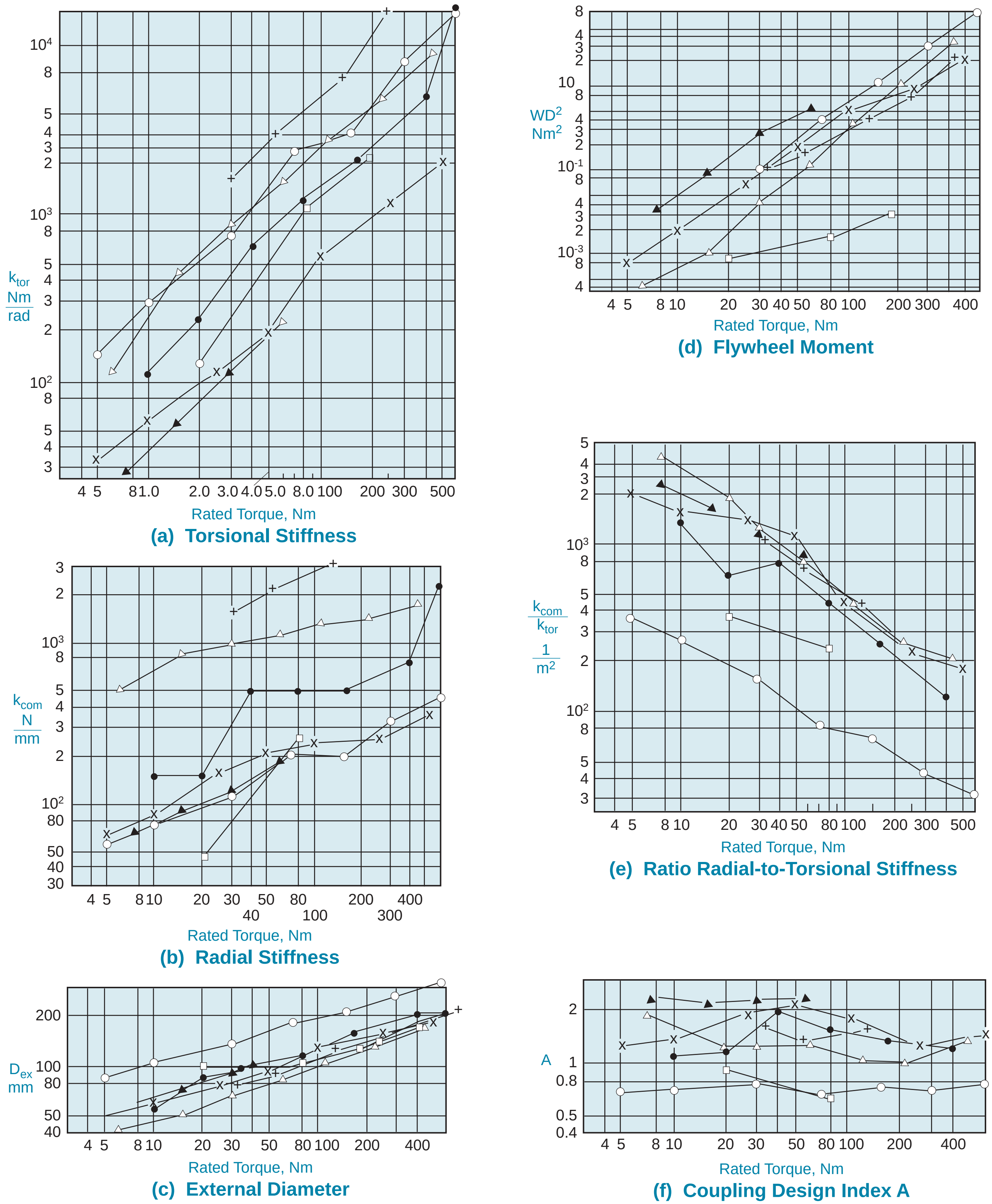


Figure 13 Basic Characteristics of Frequently Used Torsionally Flexible/Combination Purpose Couplings

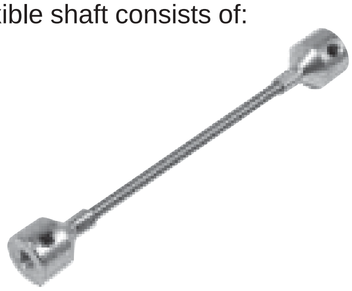
Δ - Jaw Coupling with Rubber Spider	Figure 12 (a)	□ - Rubber Disk Coupling	Not Shown
▲ - Modified Spider Coupling	Figure 12 (b)	● - Uniflex Coupling	Figure 12 (e)
○ - Toroid Shell Coupling	Figure 12 (c)	+ - Finger Sleeve Coupling	Figure 12 (d)

3.3.2.1 Miscellaneous Combination Purpose Couplings

3.3.2.1a Flexible Shafts

Flexible shafts are relatively stiff in torsion but very compliant in bending and lateral misalignments. A good example of this is in their use on automotive speedometer drives.

A flexible shaft consists of:



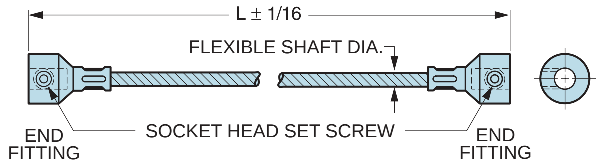


Figure 14 Flexible Shaft

a. Shaft - the rotating element comprising a center wire with several wire layers wrapped around it in alternating directions.

b. Casing - the sleeve made from metal or nonmetals to guide and protect the shaft and retain lubricants. Flexible shafts can be supplied without casing when used for hand-operated controls or intermittent-powered applications.

c. Case End Fitting - connects the casing to the housing of the driver and driven equipment.

d. Shaft End Fitting - connects the shaft to the driving and driven members. Flexible shafts as shown in the SDP/SI catalogs [4] are often substituted in place of more expensive gear trains and universal joints in applications where the load must be moved in many directions. They are extremely useful where the load is located in a remote position requiring many gear and shafting combinations.

The basic design considerations are torque capacity, speed, direction of rotation, bend radii and service conditions. Torque capacity is a function of the shaft size. Operating conditions must be considered in power drive applications such as starting torque, reversing shocks, and fluctuating loads. These conditions constitute overloads on the shaft. If they are substantially greater than the normal torque load, a larger shaft must be selected. Since, in power applications, torque is inversely proportional to speed, it is beneficial to keep the torque down, thereby reducing shaft size and cost.
Ordinarily, speeds of 1750 to 3600 rpm are recommended. However, there are applications in which shafts are operating successfully from 600 to 12,000 rpm. The general formula for determining maximum shaft speed is:

$N = (7200) / \pi d$, where N = rpm, d = shaft diameter in inches. (6)

Flexible shafting for power transmission is wound for maximum efficiency when rotating in only one direction - the direction which tends to tighten the outer layer of wires on the shaft. Direction of rotation is identified from the power source end of the shaft. Torque capacity in the opposite direction is approximately 60% of the "wind" direction. Therefore, if the power drive shaft must be operated in both directions, the reduced torque capacity will require a larger shaft than would normally be selected for operation in the wind direction.

Because flexible shafts were developed primarily as a means of transmitting power where solid shafts cannot be used, most applications involve curves. Each shaft has a recommended minimum operating radius which is determined by the shaft diameter and type. As the radius of curvature is decreased, the torque capacity also decreases and tends to shorten shaft life.

Lastly, service conditions such as temperature present no special problems to flexible shafts when operating in the -65°F to +250°F range. Plastic casing coverings are able to cover this temperature range and provide additional protection from physical abrasion as well as being oil and watertight.

3.3.2.1b Uniflex Couplings

Sometimes it is desirable if not essential that a flexible shaft coupling be as short as possible and still retain most of the features previously described. Figure 12e illustrates such a coupling, available in the SDP/SI catalog [4].

The "flexible shaft" center section consists of three separately wound square wire springs. Individual spring layers are opposingly wound to provide maximum absorption of vibration, load shock, and backlash. The hubs are brazed to the springs for maximum strength. Design data is available in Table 3 as well as in the Uniflex catalog page of the SDP/SI catalog.
The maximum torque and/or H.P. Capacity from Table 3 must be divided by the *Service Factor (S.F.)* dependent on the load character as follows:

a. Light, even load - S.F. = 1.0;
b. Irregular load without shock, rare reversals of direction - S.F. = 1.5
c. Shock loads, frequent reversals - S.F. = 2.0

TABLE 3 UNIFLEX COUPLINGS SELECTION DATA

Series Number	Max. Torque lb. in.	Horsepower Capacity* At Varying Speeds (rpm)									
		100	300	600	900	1200	1500	1800	2400	3000	3600
18	18	.03	.09	.18	.27	.36	.45	.5	.7	.9	1
25	34	.05	.15	.30	.45	.60	.75	.9	1.2	1.5	1.8
37	39	.06	.18	.36	.54	.70	.90	1	2.4	1.8	2
50	82	.13	.39	.78	1.2	1.5	2	2.3	3	3.9	4.6

*****Based on service factor of one only

Uniflex Selection Procedure:

a. Select the service factor according to the application.
b. Multiply the horsepower or torque to be transmitted by the service factor to obtain rating.
c. Select the coupling with an equivalent or slightly greater horsepower or torque than shown in Table 3.

3.3.2.1c Jaw and Spider Couplings

Jaw type couplings, Figures 12a, 12b consist of two metal hubs which are fastened to the input and output shafts (see product pages in this catalog). Trapped between the hubs is a rubber or Urethane "spider" whose legs are confined between alternating metal projections from the adjacent hubs. The spider is the wearing member and can be readily replaced without dismantling adjacent equipment. The coupling is capable of operating without lubrication and is unaffected by oil, grease, dirt or moisture. Select the proper size for your application from Table 4 and the selection instructions. The Service Factors are, essentially, the same as for the Uniflex coupling.

Jaw and Spider Type Coupling Selection Procedure:

a. Select the Service Factor according to the application.
b. Multiply the horsepower or torque to be transmitted by the service factor to obtain rating.
c. Select the coupling series from Table 4 with an equivalent or slighlty greater horsepower or torque than the calculated value in b.
d. Turn to the product section page illustrating the same coupling and make your specific selection in that number series.

TABLE 4 JAW TYPE COUPLINGS SELECTION DATA

Coupling Series Number	Rated Torque lb. in.	Service Factor	Horsepower Capacity at Varying Speeds (rpm)									
			100	300	600	900	1200	1500	1800	2400	3000	3600
035	3.5	1.0	.0056	.017	.034	.05	.067	.084	.13	.10	.17	.2
		1.5	.0037	.011	.023	.033	.045	.056	.087	.067	.113	.13
		2.0	.0028	.009	.017	.025	.033	.043	.065	.05	.025	.10
050	25.2	1.0	.04	.12	.24	.36	.48	.60	.72	.96	1.2	1.44
		1.5	.03	.08	.16	.24	.32	.40	.48	.64	.80	.96
		2.0	.02	.06	.12	.18	.24	.30	.36	.42	.60	.70
070	37.8	1.0	.06	.18	.36	.54	.72	.90	1.08	1.44	1.8	2.16
		1.5	.04	.12	.24	.36	.48	.60	.72	.96	1.2	1.44
		2.0	.03	.09	.12	.27	.36	.45	.54	.72	.90	1.08
075	75.6	1.0	.12	.36	.72	1.08	1.44	1.80	2.16	2.88	3.6	4.34
		1.5	.08	.24	.48	.72	.96	1.20	1.44	1.92	2.4	2.88
		2.0	.06	.18	.36	.54	.72	.90	1.08	1.44	1.8	2.10
090	126	1.0	.20	.60	1.2	1.8	2.4	3.0	3.6	4.8	6.0	7.2
		1.5	.13	.40	.20	1.2	1.6	2.0	2.4	3.2	4.0	4.8
		2.0	.10	.30	.60	.90	1.2	1.5	1.8	2.4	3.0	3.6

Service Factors
1.0 ____ Even Load, No Shock, Infrequent Reversing with Low Starting Torque
1.5 ____ Uneven Load, Moderate Shock, Frequent Reversing with Low Start Torque
2.0 ____ Uneven Load, Heavy Shock, Hi Peak Loads, Frequent Reversals with High Start Torque

3.3.2.1d Sleeve Type Coupling (Geargrip)

A sleeve type coupling consists of two splined hubs with a mating intermediate member of molded neoprene. Because of its construction features, it is capable of normal operation with angular shaft misalignments up to 2°.

Lubrication is not required. All parts are replaceable without disturbing adjacent equipment provided sufficient shaft length is allowed by sliding coupling hubs clear of the sleeve member during disassembly. Select the proper size for your application from Table 5 and follow the selection instructions.

Sleeve Type Coupling Selection Procedure

a. Determine motor characteristics.
b. Determine service conditions.
c. Select the coupling model with an equivalent or slightly greater horsepower than the calculated value in b in Table 5.
d. Turn to Geargrip couplings in the product section and select the specific assembly or individual components in that model number.

TABLE 5 SLEEVE TYPE COUPLINGS SELECTION DATA

Motor Torque		Motor: Normal Torque								Motor: High Torque							
Service		Normal Duty				Severe Duty				Normal Duty				Severe Duty			
Speed, rpm		3500	1750	1160	870	3500	1750	1160	870	3500	1750	1160	870	3500	1750	1160	870
	1/12	11	11	11	18	11	11	18	18	11	11	18	18	11	18	18	21
	1/8	11	11	18	18	11	18	18	21	11	18	18	21	11	18	21	31
	1/6	11	18	18	21	11	18	21	21	11	18	21	21	18	21	31	31
H.P.	1/4	11	18	21	31	18	21	31	31	18	21	31	31	18	31	31	31
	1/3	18	21	31	31	18	31	31	31	18	31	31	31	21	31	31	31
	1/2	18	31	31	31	21	31	31	31	21	31	31	31	31	31	31	
	3/4	21	31	31	31	31	31			31	31			31	31		
	1	31	31	31		31	31			31	31			31	31		

Service Conditions

Normal Duty
- speed not exceeding 3600 rpm
- operation less than 10 hours per day
- infrequent stops and starts
- no heavy, pulsating load
- no mechanical or electrical clutch

Severe Duty
- speeds from 3600 to 5000 rpm
- operation runs more than 10 hours per day
- frequent starts and stops
- heavy, pulsating load
- mechanical or electrical clutch

Other types of couplings are also available and are fully described along with technical specifications in the SDP/SI catalogs dealing with couplings [4].

References

[1] Rivin, E.I., *Stiffness and Damping in Mechanical Design*, 1999, Marcel Dekker Inc.

[2] Baranyi, S.J., "The Analytical Design of Universal Joints", Design News, 1969, Sept. 1

[3] Rivin, E.I., "Design and Application Criteria for Connecting Couplings", 1986, ASME Journal of Mechanisms, Transmissions, and Automation in Design, vol. 108, pp. 96-105 (this article is fully reprinted in [1])

[4] Stock Drive Products/Sterling Instrument, *Catalog D790, Handbook of Inch Drive Components* and *Catalog D785, Handbook of Metric Drive Components* or their current catalogs.